

## DOCTORAL THESIS

### Combinatorial design via association scheme

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# Combinatorial Design via Association Scheme

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# Abstract

The main contribution of the thesis is to give some new theories and applications of association scheme and other combinatorial structures in design theory. In experimental design theory and algebraic combinatorics, the association scheme is an important subject. It has applications in Partially Balanced Incomplete Block (PBIB) designs, Coding Theory and Finite Geometry, and it is also an important design itself. In this thesis, some new applications of association schemes in studying orthogonal arrays and frequency squares are found and two families of association schemes are generalized from  $(t, m, s)$ -nets.

In the thesis, the association scheme is used to study orthogonal arrays and symmetric schematic orthogonal arrays are found. This is a partial solution of an open problem about schematic orthogonal array by Hedayat, Sloane and Stufken in their book *Orthogonal Arrays*. The solution of Hedayat's problem is given for symmetric orthogonal arrays with strength two, and some cases of asymmetric orthogonal array with strength two are discussed as well. It is the first contribution of the thesis.

A frequency square is a generalization of a latin square. Its construction and application is an interesting problem. In the thesis, a relation between frequency squares and association schemes is found, and some methods of constructing pairs of symmetric orthogonal frequency square via association schemes are given. This is the second contribution of the thesis.

Since the association scheme is an important subject itself, the construction of it is also interesting. In this thesis, some construction methods are given. The idea of construction by parameter amendment is given. Some association schemes are constructed from BIB designs and some special graphs. The construction of 2-cover antipodal association schemes is studied, and some of their important algebraic properties are found. Based on these algebraic properties of the scheme, the construction of partially balanced block design is discussed. The study of the construction and algebraic properties of 2-cover antipodal association scheme is the third contribution of this thesis.

This thesis also discusses the relationship between combinatorial design and digi-

tal nets. Two methods for constructing association schemes from some  $(t, m, s)$ -nets in base  $b$  are given. By the methods, the family of 2-class latin-square-type association scheme and the family of 3-class cubic association scheme have been generalized. A combination method for the digital net is given by the idea of mending coordinates of the points and a known construction method is improved as well. This is the fourth contribution of the thesis.

**Keywords:** schematic orthogonal array, Hamming distance, association scheme, Hadamard matrix, latin square, frequency square, difference matrix, BIB design, PBIB design, adjacency matrix, intersection matrix,  $(t, m, s)$ -net in base  $b$ .

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