

DOCTORAL THESIS

Continuous methods in optimization and its application in discriminant analysis

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Continuous Methods in Optimization and Its Application in Discriminant Analysis

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Abstract

Continuous methods in optimization have increasingly attracted attention recently. In the first part of this thesis, we develop three continuous methods for specific optimization problems resulting respectively from the unconstrained optimization, the extreme and interior eigenvalue problems and the Linear Discriminant Analysis (LDA) with sparsity consideration in pattern recognition. The first method defines an ordinary differential equation by combining the Newton direction and the negative gradient in a special way for solving the unconstrained minimization problem. We show that for a $C^2(\mathbb{R}^n)$ objective function, our method converges globally to a connected subset of the stationary points under some mild conditions, and converges globally to a single stationary point for a real analytic function. The method reduces to the *continuous Newton method* when the Hessian matrix of the objective function is uniformly positive definite. The second continuous method generalizes the continuous method in [63] (Golub and Liao) for solving extreme and interior eigenvalue problems. The generalized method is proved to converge faster and almost globally, and it shares many surprisingly analogous properties with the *Rayleigh quotient gradient flow*. Based on the gradient flow, our third continuous method solves a problem from the Linear Discriminant Analysis, one of the most popular approaches in pattern recognition. In this problem, the objective function involves a non-differentiable term and the constraint is the *compact Stiefel manifold*. For each method, we provide a sufficient numerical testing and show their advantages.

The second part focuses on the generalized Foley-Sammon transform (GFST [53, 69], or a trace ratio model [148]) and its regularization (RGFST) for *undersampled problems*. The GFST is an important model in LDA and begins to receive interests recently. We characterize the global solutions set of the resulted optimization problem and apply the iteration method proposed in [148] to solve it. The global convergence has been proved and fast convergence was observed numerically in [148], but no theoretical analysis on the convergence rate is carried out. We first prove that the iteration is quadratically convergent under a mild condition, and show also that the local quadratic convergence holds in two special inexact computations; moreover,

under no assumption, we prove its local superlinear convergence. To reduce the storage requirement and computational complexity of the iteration for the undersampled problem, we establish an equivalent reduced model of the RGFST based on which an efficient approach is proposed. Practical implementations including complexity and storage of our method are discussed, and experimental results on several real world data sets reported indicate the efficiency of the algorithm and the advantages of the RGFST in classification.

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