DOCTORAL THESIS

Projected adaptive-to-model tests for regression models

Tan, Falong

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Abstract

This thesis investigates Goodness-of-Fit tests for parametric regression models. With the help of sufficient dimension reduction techniques, we develop adaptive-to-model tests using projection in both the fixed dimension settings and the diverging dimension settings.

The first part of the thesis develops a globally smoothing test in the fixed dimension settings for a parametric single index model. When the dimension p of covariates is larger than 1, existing empirical process-based tests either have non-tractable limiting null distributions or are not omnibus. To attack this problem, we propose a projected adaptive-to-model approach. If the null hypothesis is a parametric single index model, our method can fully utilize the dimension reduction structure under the null as if the regressors were one-dimensional. Then a martingale transformation proposed by Stute, Thies, and Zhu (1998) leads our test to be asymptotically distribution-free. Moreover, our test can automatically adapt to the underlying alternative models such that it can be omnibus and thus detect all alternative models departing from the null at the fastest possible convergence rate in hypothesis testing. A comparative simulation is conducted to check the performance of our test. We also apply our test to a self-noise mechanisms data set for illustration.

The second part of the thesis proposes a globally smoothing test for parametric single-index models in the diverging dimension settings. In high dimensional data analysis, the dimension p of covariates is often large even though it may be still small compared with the sample size n. Thus we should regard p as a diverging number as n goes to infinity. With this in mind, we devolop an adaptive-to-model empirical process as the basis of our test statistic, when the dimension p of covariates diverges to infinity as the sample size n tends to infinity. We also show that the martingale transformation proposed by Stute, Thies, and Zhu (1998) still work in the diverging dimension settings. The limiting distributions of the adaptive-to-model empirical process under both the null and the alternative are discussed in this new situation. Simulation examples are conducted to show the performance of this test when p grows with the sample size n.

The last Chapter of the thesis considers the same problem as in the second part. Bierens's (1982) first constructed tests based on projection pursuit techniques and obtained an integrated conditional moment (ICM) test. We notice that Bierens's (1982) test performs very badly for large p, although it may be viewed as a globally smoothing test. With the help of sufficient dimension techniques, we propose an adaptive-to-model integrated conditional moment test for regression models in the diverging dimension setting. We also give the asymptotic properties of the new tests under both the null and alternative hypotheses in this new situation. When p grows with the sample size n, simulation studies show that our new tests perform much better than Bierens's (1982) original test.

Keywords: Adaptive-to-model test; Empirical process; Globally smoothing test; Integrated conditional moment test; Martingale transformation; Projection pursuit; Single-index models; Sufficient dimension reduction.

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