DOCTORAL THESIS

From a linear birth-growth model to insurance risk models with applications to finance

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Abstract

The main goal of the thesis is to study certain problems in stochastic geometry, insurance risk theory and finance by using Markov processes as well as renewal theory and martingales.

Our first goal is to consider the linear birth-growth model in stochastic geometry. Let Φ be a space-time Poisson process on $\mathbb{R}\times[0,\infty)$ with intensity measure $\mathrm{d}x\lambda(t)\mathrm{d}t$, where $\lambda(\cdot)$ is integrable and such that for all $t>0$, $0<\Lambda(t):=\int_0^t \lambda(y)\mathrm{d}y<\infty$. Denote the points of Φ by $\{(x_i;t_i), i\in\mathbb{N}\}$. Suppose for each $i\in\mathbb{N}$ an interval starts to grow in $\mathbb{R}$ from x_i with constant speed in both directions at time t_i. The union of growing random intervals thus generated will cover any finite interval $(0,L)$ after an almost surely finite random time. In this thesis a Markov process approach is used to establish the exact and limiting distributions and strong limit theorems for the time of complete coverage of a sufficiently long interval.

Another goal is to consider various risk processes in actuarial mathematics. The classical risk model is generalized to more general ones such as risk processes with reserve-dependent premium, diffusion perturbed processes and models that allow for stochastic return on investments as well as perturbation by diffusion. These processes can be formulated as piecewise deterministic Markov processes or jump-diffusion processes. Markov processes as well as renewal theory and martingales are used to study the distributions of ruin time, the occupation time, the surplus prior to ruin and the deficit at ruin. Spectrally negative Lévy processes are also used to study certain risk models.

Finally, applications in mathematical finance, such as the path properties of stock price, the valuation of perpetual American options and the distribution of optimal exercise time, are studied.

Keywords: completion time; finance; first passage time; integro-differential equation; inhomogeneous Poisson process; Johnson-Mehl model; last passage time; linear birth-growth model; martingale; optimal exercise time; perpetual American option; renewal theory; risk theory; ruin probability; spectrally negative Lévy process; stock price; surplus at ruin; surplus prior to ruin; time of ruin.

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Contents