

DOCTORAL THESIS

From a linear birth-growth model to insurance risk models with applications to finance

Yin, Chuancun

Date of Award:
2002

[Link to publication](#)

General rights

Copyright and intellectual property rights for the publications made accessible in HKBU Scholars are retained by the authors and/or other copyright owners. In addition to the restrictions prescribed by the Copyright Ordinance of Hong Kong, all users and readers must also observe the following terms of use:

- Users may download and print one copy of any publication from HKBU Scholars for the purpose of private study or research
- Users cannot further distribute the material or use it for any profit-making activity or commercial gain
- To share publications in HKBU Scholars with others, users are welcome to freely distribute the permanent URL assigned to the publication

**From a Linear Birth-Growth Model
to Insurance Risk Models with
Applications to Finance**

YIN Chuancun

**A thesis submitted in partial fulfilment of the requirements
for the degree of
Doctor of Philosophy**

March 2002

Hong Kong Baptist University

Abstract

The main goal of the thesis is to study certain problems in stochastic geometry, insurance risk theory and finance by using Markov processes as well as renewal theory and martingales.

Our first goal is to consider the linear birth-growth model in stochastic geometry. Let Φ be a space-time Poisson process on $\mathbb{R} \times [0, \infty)$ with intensity measure $dx\lambda(t)dt$, where $\lambda(\cdot)$ is integrable and such that for all $t > 0$, $0 < \Lambda(t) := \int_0^t \lambda(y)dy < \infty$. Denote the points of Φ by $\{(x_i; t_i), i \in \mathbb{N}\}$. Suppose for each $i \in \mathbb{N}$ an interval starts to grow in \mathbb{R} from x_i with constant speed in both directions at time t_i . The union of growing random intervals thus generated will cover any finite interval $(0, L)$ after an almost surely finite random time. In this thesis a Markov process approach is used to establish the exact and limiting distributions and strong limit theorems for the time of complete coverage of a sufficiently long interval.

Another goal is to consider various risk processes in actuarial mathematics. The classical risk model is generalized to more general ones such as risk processes with reserve-dependent premium, diffusion perturbed processes and models that allow for stochastic return on investments as well as perturbation by diffusion. These processes can be formulated as piecewise deterministic Markov processes or jump-diffusion processes. Markov processes as well as renewal theory and martingales are used to study the distributions of ruin time, the occupation time, the surplus prior to ruin and the deficit at ruin. Spectrally negative Lévy processes are also used to study certain risk models.

Finally, applications in mathematical finance, such as the path properties of stock price, the valuation of perpetual American options and the distribution of optimal exercise time, are studied.

Keywords: completion time; finance; first passage time; integro-differential equation; inhomogeneous Poisson process; Johnson-Mehl model; last passage time; linear birth-growth model; martingale; optimal exercise time; perpetual American option; renewal theory; risk theory; ruin probability; spectrally negative Lévy process; stock price; surplus at ruin; surplus prior to ruin; time of ruin.

AMS 2000 Subject Classification: 60G55; 60J25; 60F05; 60F15; 60D05; 91B30; 60J70; 62P05; 60G40; 60G51.

Contents

Declaration	i
Abstract	ii
Acknowledgements	iv
Contents	v
1 Introduction	1
1.1 Overview	1
1.2 Outline of the thesis	1
2 The time of completion of a linear birth-growth model	6
2.1 Introduction	6
2.2 Laplace transform of the completion time	7
2.3 Exact distribution	9
2.4 Limiting distributions	11
2.5 Strong limit theorems	14
2.6 An alternative approach	16
3 The occupation times, the first exit times and the ruin times for a risk process with reserve-dependent premium	18
3.1 The occupation times	18
3.1.1 Introduction	18
3.1.2 Preliminaries	20
3.1.3 Occupation times	23
3.1.4 The classical model	26
3.2 The first exit time and the ruin time	29

3.2.1	Introduction	29
3.2.2	Application to the classical model	32
3.2.3	Application to the Embrechts-Schmidli model	39
4	The time of ruin, the surplus prior to ruin and the deficit at ruin for the classical risk process perturbed by diffusion	44
4.1	The distributions of the time of ruin, the surplus prior to ruin and the deficit at ruin	44
4.1.1	Introduction	44
4.1.2	Integro-differential equation	47
4.1.3	The joint distributions	51
4.2	Asymptotic behaviour of the surplus prior to and at ruin	55
4.2.1	Introduction	55
4.2.2	The exact expressions	57
4.2.3	Lighted-tailed cases	61
4.2.4	Heavy-tailed cases	61
4.2.5	Intermediate cases	63
5	Diffusion perturbed risk processes with stochastic return on investments	66
5.1	Introduction	66
5.2	Basic theory	67
5.3	The model	68
5.4	Integro-differential equations	71
5.5	Risk model with interest force	74
6	Passage times for a spectrally negative Lévy process with applica-	

tions to risk theory	81
6.1 Introduction	81
6.2 No Gaussian component	83
6.3 General case	86
6.4 Applications to risk theory	90
7 Applications to finance	93
7.1 Path properties of stock price	93
7.2 Pricing perpetual American options.....	96
7.3 Closed form solutions with Erlang distributed jumps.....	102
Bibliography	106
Curriculum Vitae	114