

DOCTORAL THESIS

Digital nets and sequences for quasi-Monte Carlo methods

Hong, Hee Sun

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Digital Nets and Sequences for Quasi-Monte Carlo Methods

HONG Hee Sun

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Principal Supervisor: Prof. Fred. J. Hickernell

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Abstract

Quasi-Monte Carlo methods are a way of improving the efficiency of Monte Carlo methods. Digital nets and sequences are one of the low discrepancy point sets used in quasi-Monte Carlo methods. This thesis presents the three new results pertaining to digital nets and sequences: implementing randomized digital nets, finding the distribution of the discrepancy of scrambled digital nets, and obtaining better quality of digital nets through evolutionary computation. Finally, applications of scrambled and non-scrambled digital nets are provided.

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