DOCTORAL THESIS

Geometric structures of eigenfunctions with applications to inverse scattering theory, and nonlocal inverse problems

Cao, Xinlin

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Abstract

Inverse problems are problems where causes for desired or an observed effect are to be determined. They lie at the heart of scientific inquiry and technological development, including radar/sonar, medical imaging, geophysical exploration, invisibility cloaking and remote sensing, to name just a few.

In this thesis, we focus on the theoretical study and applications of some intriguing inverse problems. Precisely speaking, we are concerned with two typical kinds of problems in the field of wave scattering and nonlocal inverse problem, respectively.

The first topic is on the geometric structures of eigenfunctions and their applications in wave scattering theory, in which the conductive transmission eigenfunctions and Laplacian eigenfunctions are considered. For the study on the intrinsic geometric structures of the conductive transmission eigenfunctions, we first present the vanishing properties of the eigenfunctions at corners both in $\mathbb{R}^2$ and $\mathbb{R}^3$, based on microlocal analysis with the help of a particular type of planar complex geometrical optics (CGO) solution. This significantly extends the previous study on the interior transmission eigenfunctions. Then, as a practical application of the obtained geometric results, we establish a unique recovery result for the inverse problem associated with the transverse electromagnetic scattering by a single far-field measurement in simultaneously determining a polygonal conductive obstacle and its surface conductive parameter. For the study on the geometric structures of Laplacian eigenfunctions, we separately discuss the two-dimensional case and the three-dimensional case. In $\mathbb{R}^2$, we introduce a new notion of generalized singular lines of Laplacian eigenfunctions, and carefully investigate these singular lines and the nodal lines. The studies reveal that the intersecting angle between two of those lines is closely related to the vanishing order of the eigenfunction at the intersecting point. We provide an accurate and comprehensive quantitative characterization of the relationship. In $\mathbb{R}^3$, we study the analytic behaviours of Laplacian eigenfunctions at places where nodal or generalized singular planes intersect, which is much more complicated. These theoretical findings are original and of significant interest in spectral theory. Moreover, they are applied directly to some physical problems of great importance, including the

inverse obstacle scattering problem and the inverse diffraction grating problem. It is shown in a certain polygonal (polyhedral) setup that one can recover the support of the unknown scatterer as well as the surface impedance parameter by finitely many far-field patterns.

Our second topic is concerning the fractional partial differential operators and some related nonlocal inverse problems. We present some prelimilary knowledge on fractional Sobolev Spaces and fractional partial differential operators first. Then we focus on the simultaneous recovery results of two interesting nonlocal inverse problems. One is simultaneously recovering potentials and the embedded obstacles for anisotropic fractional Schrödinger operators based on the strong uniqueness property and Runge approximation property. The other one is the nonlocal inverse problem associated with a fractional Helmholtz equation that arises from the study of viscoacoustics in geophysics and thermoviscous modelling of lossy media. We establish several general uniqueness results in simultaneously recovering both the medium parameter and the internal source by the corresponding exterior measurements. The main method utilized here is the low-frequency asymptotics combining with the variational argument. In sharp contrast, these unique determination results are unknown in the local case, which would be of significant importance in thermo- and photoacoustic tomography.

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