

MASTER'S THESIS

Topics in Banach space theory

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Topics in Banach Space Theory

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for the degree of

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Abstract

In this thesis we describe some topics in classical and modern Banach space theory, ranging from bases and geometrical properties to various aspects of bounded linear operators. The choice of topics reflects my particular interests, and the presentation is complemented by a number of my own research contributions.

In the first chapter, we start by presenting some examples of Banach spaces, followed by a brief introduction to linear operators and a section on geometrical properties. We then study various aspects of Schauder bases. Finally, a new type of basis is presented: it is motivated by my approach to transforming linear operator equations in l^∞ into a Fredholm integral equation of the first kind.

In the second chapter, several conditions which are equivalent to the approximation property are discussed. We also analyse the finite-dimensional expansion of the identity and provide, at the end of the chapter, several counterexamples about the approximation property.

The third chapter deals with three topics on bounded linear operators. The first one concerns the Laurent series and a new decomposition of operators based on Laurent series. We then introduce a special type of basis decomposition of elements. This includes the canonical basis, the Laurent Theorem and the Fourier series. Lastly, we look at approximate eigenvalues of bounded linear operators and their characterisation.

In the fourth chapter, we discuss three more advanced topics in Banach space theory, starting with C_0 -semigroups. Here, we focus on the characterization of their infinitesimal generators and on three exponential formulas. We also explain how they are related to the Laplace transform. The next topic is C^* -algebras. We start with their basic properties and then turn to positive linear functionals and the Gelfand-Naimark-Segal Theorem. Finally, we study quasi-nilpotent operators: we describe important properties, including their quasi-nilpotent parts, and illustrate them by numerous examples.

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