DOCTORAL THESIS

Solving optimization problems with generalized orthogonality constraints

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Abstract

This thesis focuses on optimization problems with generalized orthogonality constraints, which may also contains linear equality constraints. These problems appear in many areas, such as machine learning, signal processing, computer vision and so on.

Many problems in this form are NP hard. One challenge posed by generalized orthogonality constraints is local minimizers loaded by nonconvex constraints. Moreover, the generalized orthogonality constraints are numerically expensive to preserve during iterations.

This thesis is mainly divided into two parts. The first part is focused on solving generalized orthogonality constrained optimization problems with differentiable objective functions. For this class of optimization problems, a generalized gradient flow is proposed, which is contained on the constraints set if the initial condition satisfies generalized orthogonality constrains. The week convergence of the generalized gradient flow is given. A discrete iterative scheme is also proposed to make the gradient flow method computable. In addition, we analyze the relationship between our discrete iteration scheme and some existing constraint preserving methods, and the relationship between our discrete iteration scheme and the inexact forward-backward method, respectively. Several problems which also can be solved by the generalized gradient flow are given. Furthermore, we also propose an optimal gradient flow by analyzing the first order optimality condition.

The second part of this thesis is devoted to study of the generalized orthogonality constrained optimization problems with nondifferentiable objective functions. An approximate augmented Lagrangian method is used to deal with this class of problems. The global convergence is presented. We also extend the proximal alternating linearized minimization method (EPALM) to deal with the generalized orthogonality constraints appeared in the subproblem of the approximate augmented Lagrangian method. Moreover, to accelerate the EPALM method, an inertial proximal alternat-

ing linearized minimization method (IPALM) is proposed to deal with unconstrained nonconvex, nonsmooth problems with coupled objective functions.

Keywords: Generalized Orthogonality Constraints; Stiefel Manifold; Tangent Space; Gradient Flow; Approximate Augmented Lagrangian Method; Proximal Alternating Linearized Minimization Method

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