

DOCTORAL THESIS

Moving finite element methods for phase-field models of solidification

Wang, Heyu

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Moving Finite Element Methods for Phase-field Models of Solidification

WANG Heyu

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for the degree of
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Principal Supervisor: Prof. TANG Tao

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Abstract

The phase-field approach is one of the main choices for simulating phenomena in solidification and other related systems. Since the solutions of the phase-field equations have large solution variations, adaptive grid methods have become important in their numerical simulations. This thesis is mainly concerned with the applications of one of the adaptive grid methods, namely, the so called moving mesh method (or r -adaptive method), to the finite element discretization of the phase-field equations arising from the solidification problem.

We begin by briefly discussing various models for phase-field approaches for solidification. We then present a simple moving mesh method for solving phase-field equations with isotropy surface energy. The mesh redistribution is realized by solving an elliptic boundary control problem. Moreover, an efficient alternating Crank-Nicolson time discretization scheme is developed for solving the nonlinear system resulting from a finite element approximation to the phase-field equations. The efficiency of the method is demonstrated by numerical experiments.

For a simple model problem, some existing moving mesh solutions do not agree with the numerical solutions on very fine fixed mesh. From our numerical studies, it is concluded that for the phase-field equations the numerical solutions are sensitive to the starting mesh and the monitor function. The present numerical evidences indicate that the under-resolution of the interface curvature maybe the source of the qualitatively error in the numerical solutions. This is mainly due to the use of some inappropriate starting meshes. Our computations suggest that the Delaunay mesh is in general a good choice of the starting mesh.

Based on the above results, we present an efficient moving finite element algorithm for the phase field model of dendritic growth in both two and three dimensions cases. The nodes redistribution was improved by a recently developed nonlinear multi-grid algorithm to make the 3D simulations possible. With a particularly designed monitor function, the quality of the redistributed mesh grids is improved significantly. Consequently, the numerical results with fewer degrees of freedom is comparable with the results on finer uniform meshes. It is illustrated by the numerical experiments that the tip velocity obtained by our numerical algorithm is in good agreement with the published ones. Due to the high efficiency of the proposed algorithm, both 2D and 3D simulations can be carried out on standard desktop computers.

We remark that although the monitor functions proposed in this work are specially designed for the dendritic growth problem, the basic idea of the approach seems applicable for designing monitor functions for more general problems involving interfaces.

Keywords: Phase field model, Moving mesh method, Crank-Nicolson scheme, Numerical sensitivity, Finite element method, Dendritic growth.

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