

## DOCTORAL THESIS

### Non-negative matrix factorization for face recognition

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# Non-negative Matrix Factorization for Face Recognition

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# Abstract

Face recognition has received significant attention over the past three decades for its wide range of commercial and law enforcement applications, and a number of algorithms have been developed. Among them, Principal Component Analysis (PCA) (i.e. Eigenface) has been proven to be a successful face-based approach for this problem [1–7].

However, the traditional Eigenface approach has its limitations. First, PCA representation has a poor discriminatory ability even though it gives a very good representation of the images. Secondly, PCA basis images do not yield intuitive visual meaning. Furthermore, this approach is based on extracting global face features, so the problem of occlusions is difficult to handle.

Recently, a new technique for obtaining a linear representation of data has been proposed [8–21]. This new method, called Non-negative Matrix Factorization (NMF), differs from previous methods by the usage of non-negativity constraints. It approximately factorizes the initial data matrix, which represents the whole database, into two non-negative matrix factors and consequently produces a parts-based representation of images because it allows only additive, not subtractive, combinations of basis images.

In face recognition problem, we project all the faces into this NMF space and obtain their corresponding feature vectors. Comparison is performed by calculating the distance between these vectors. Although there exist many distance measures, we are able to find only few attempts to propose, compare and use distance measures [22, 23] for NMF-based face recognition to achieve better recognition results.

In this thesis we conducted a thorough review of distance measures and also proposed two new non-negative vector similarity coefficient-based (NVSC) distance measures that we are advocating for use in NMF-based face recognition. Our experiments show that these new distance measures are always among the best distance measures with respect to different image databases and at different settings.

We have used the Principal Component Analysis (i.e. Eigenface) combined with common distance measures for a direct comparison, and the experimental result also

supports the conclusion that our new distance measures combined with NMF can achieve a better performance for identifying the probe images in database.

Another two crucial factors in NMF are the structured initialization and determination of the number of basis images. We proposed using the clustering method to produce a structured initialization for NMF. In proposing this initialization strategy, we also arrive at a new efficient way of choosing the number of NMF basis images. The corresponding performance is very encouraging. The computational complexity and the recognition result for NMF algorithm are both improved.

Finally, this thesis also explores the modification of NMF algorithm for face recognition. Since its training procedure is implemented in an unsupervised way, the discrimination information in the training set is not exploited efficiently to boost the classification capability. In this thesis we introduce an LDA-based Non-negative Matrix Factorization algorithm which is a new variation to NMF. To take advantage of more information in the training images, we add the Fisher Linear Discriminant into the NMF algorithm, which will lead to base vectors and weight vectors with more discrimination information. Under a mild condition, the update rule guarantees the non-negativity for all the coefficients and thus preserves the intuitive meaning for the base vector and weight vector. Since this algorithm encodes discrimination information for face recognition, it should improve the result for classification. The experimental result also supports the conclusion that the new algorithm can achieve a better performance in face recognition.

Each method developed in this thesis has been extensively evaluated using publicly available databases such as ORL, CMU AMP, CBCL, CBCL2, FERET, YaleB and CMU PIE databases. Comparison between our proposed algorithms and related traditional algorithms are performed and reported.

In short, the major contributions of this thesis are summarized as follow:

- A comprehensive survey on the recognition performance of different distance measures is conducted to find the best distance for the NMF approach;
- A scheme is proposed to find a structured initialization and suggest the number of bases for the NMF algorithm. The experimental result shows this strategy can speed up the convergence of NMF algorithm and improve the performance

for face recognition;

- An LDA-based Non-negative Matrix Factorization algorithm, which integrates the Fisher Linear Discriminant into the NMF algorithm, is proposed to improve the result for face classification.

**Keywords:** Face recognition, Non-negative Matrix Factorization, Distance measures, Principal Component Analysis, Eigenface, Fisherface, Clustering, Initialization.

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