

DOCTORAL THESIS

Incidence coloring: origins, developments and relation with other colorings

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Date of Award:
2007

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Incidence Coloring: Origins, Developments and Relation with Other Colorings

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A thesis submitted in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy

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Hong Kong Baptist University

August 2007

Abstract

Incidence coloring of a graph G is a mapping from the set of incidences to a color-set C such that adjacent incidences of G are assigned distinct colors. From the development of incidence coloring, we figure out some weaknesses in the earlier proofs and remedy them by establishing a relationship between edge coloring and incidence coloring. Also, we link up the incidence chromatic number with the chromatic number of the square of a graph.

To improve the lower bound of incidence chromatic number, we provide four sufficient conditions for the graphs that are not $(\Delta+1)$ -incidence colorable. In addition, the proofs of incidence coloring of complete bipartite graphs (*Discrete Math.*, **122** (1993), 51-58) as well as outerplanar graphs (*Discrete Math.*, **256** (2002), 397-405) seem to be incorrect. We provide alternative proofs of them by adopting a totally different point of view. Finally, we determine the incidence chromatic number of K_4 -minor free graphs and cubic Halin graphs and show the results are best possible.

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