DOCTORAL THESIS

Some optimalities of uniform designs and projection uniform designs under multi-factor models

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Some Optimalities of Uniform Designs and Projection Uniform Designs under Multi-Factor Models

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Preface

In today's technical and industrial applications, one may face a system with high dimensional input and nonlinear relationship (response) between input and output. Usually, we want to model the complex response by a fitting model based on input values and output ones. Of course, the quality of the fit depends on choices (design) of input values. Due to the complexity of the system, a "space filling" design is needed (e.g., see Fang and Hickernell, 1995, and Bate, Buck, Riccomagno and Wynn, 1996). In the literature some such designs have been proposed. Following are two frequently used designs:

- the uniform design;
- the Latin hypercube sample.

The former supplies global uniform points over the experimental domain in the sense of some model-independent measure of uniformity. While the latter and its versions provide projective uniform sample points in the sense that these points stratify low dimensional margins simultaneously.

The uniform design and the Latin hypercube sample have advantages in computer experiments and their uniformities play important roles in getting these optimalities. Motivated by these facts, this thesis investigates the usefulness of uniformity in experimental designs. The research mainly concentrates on the two kinds of uniformities mentioned above:

- global uniformity;
- projective uniformity.

The measures of usefulness are taken as:

- the criteria in decision theory for approximately linear models and nonparametric models;
- the criteria in optimum design theory for Fourier models and wavelet models.

Since the uniform design is a kind of discrete approximation to the uniform design measure (uniform distribution) over the experimental domain and the most

tables of the uniform designs (see Fang, 1994) are generated by the good lattice designs, the thesis deals with optimalities of the uniform design measure and the lattice design. Based on the projective uniformity of the Latin hypercube sample and its versions, the projective uniform design is introduced and its optimalities in experimental designs are studied. The whole thesis is organized as follows:

Chapter 1 gives backgrounds and concepts of the uniform design and the Latin hypercube sample, and introduces optimalities of them in computer experiments, which are the motivations of the thesis. Finally, a summary of the thesis is given.

Chapter 2 is to make necessary preparations for later use. Some contents related to optimal design theory, product model and wavelet theory are provided.

Chapter 3 researches optimalities of the uniform design measure for a nonparametric model. Under a framework of decision theory, we prove that the uniform design measure is an admissible minimax design and the best design among a reasonable design class. These optimalities and robustness of the uniform design measure obtained by Wiens (1991) for an approximately linear model also show optimalities of the uniform design in experimental designs. This is because the uniform design is a kind of discrete approximation to the uniform design measure.

Chapter 4 studies optimalities of the lattice design for interaction Fourier models. Riccomagno, Schwabe and Wynn (1995) obtained D-optimal lattice designs for certain interaction Fourier models. We extend their results to general interaction Fourier models. These optimalities of the lattice design show at least in part those of the uniform design as most tables of the uniform designs are generated by the good lattice designs.

Chapter 5 extends the results of D-optimal designs obtained by Herzberg and Traves (1994) for Haar wavelet models to interactive Haar type wavelet models. We prove that a design is D-optimal if and only if it is projective uniform. This result also shows that the sufficient condition obtained by Herzberg and Traves (1994) is also necessary.

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