MASTER'S THESIS

On a hybrid finite element with weak Kirchhoff assumption

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On a Hybrid Finite Element with Weak Kirchhoff Assumption

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Abstract

The governing equation of classical thin plate theory is described by a biharmonic equation of the transversal displacement w such that

$$\Delta^2 w = \frac{1}{D} q(x,y)$$

where D is the bend stiffness of the plate and $q(x,y)$ is the transversal load per surface on the plate. This equation is based on the Kirchhoff-Love(K-L) assumptions:

$$(1)\ \sigma_{zz} = 0, \quad (2)\ \varepsilon_{zz} = 0 \quad \text{and} \quad (3)\ \varepsilon_{xz} = 0,\ \varepsilon_{yz} = 0,$$

where σ_{ij} and ε_{ij} $(i, j = x, y, z)$ are the stress and strain tensors respectively. The third assumption is called K-L hypothesis which was found contradictions to some fundamental mechanical principle. Many researches on the thin plate approached as a result of Mindlin plate with small thickness found that the "Locking" phenomenon (i.e. the result of the Mindlin plate does not converge to that of the thin plate) occurs. In this thesis, however, is treated as its weak form during the deduction of finite element formulation. The approach is a consequence of Hellinger-Reissner variational principle for the Mindlin plate when the thickness goes to zero and then reduce the contradiction. Based on the weak-Kirchhoff assumption, the hybrid finite element method given in the thesis can avoid the locking phenomenon. It is shown that the K-L condition will tend to zero in order of $O(h^2)$ when h goes to zero. Besides, a least order WKT-9 finite element is found under the mathematical analysis of the weak-Kirchhoff element. Based on our knowledge this is the only least order finite element for thin plate, which satisfies the BB-condition.

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