

MASTER'S THESIS

On a hybrid finite element with weak Kirchhoff assumption

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**On a Hybrid Finite Element with
Weak Kirchhoff Assumption**

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Abstract

The governing equation of classical thin plate theory is described by a biharmonic equation of the transversal displacement w such that

$$\Delta^2 w = \frac{1}{D} q(x, y)$$

where D is the bend stiffness of the plate and $q(x, y)$ is the transversal load per surface on the plate. This equation is based on the Kirchhoff-Love(K-L) assumptions:

$$(1) \sigma_{zz} = 0, \quad (2) \varepsilon_{zz} = 0 \quad \text{and} \quad (3) \varepsilon_{xz} = 0, \quad \varepsilon_{yz} = 0,$$

where σ_{ij} and ε_{ij} ($i, j = x, y, z$) are the stress and strain tensors respectively. The third assumption is called K-L hypothesis which was found contradictions to some fundamental mechanical principle. Many researches on the thin plate approached as a result of Mindlin plate with small thickness found that the "Locking" phenomenon (i.e. the result of the Mindlin plate does not converge to that of the thin plate) occurs. In this thesis, however, is treated as its weak form during the deduction of finite element formulation. The approach is a consequence of Hellinger-Reissner variational principle for the Mindlin plate when the thickness goes to zero and then reduce the contradiction. Based on the weak-Kirchhoff assumption, the hybrid finite element method given in the thesis can avoid the locking phenomenon. It is shown that the K-L condition will tend to zero in order of $O(h^2)$ when h goes to zero. Besides, a least order WKT-9 finite element is found under the mathematical analysis of the weak-Kirchhoff element. Based on our knowledge this is the only least order finite element for thin plate, which satisfies the BB-condition.

Table of Contents

Declaration	i
Abstract	ii
Acknowledgement	iii
Table of Contents	iv
List of Figures	vi
List of Tables	vii
1 Introduction	1
2 Mathematical Theory of Finite Element Method	4
2.1 Variational Problem	5
2.2 Saddle point Problem	12
3 Linear Elasticity and Generalized Variational Principles	16
3.1 Classical Problem of Linear Elasticity	16
3.2 Constitutive Law and Energy Density	18
3.3 Variational Principles in Linear Elasticity	21
4 Mathematical Theory of Thin Plate	23
4.1 Governing Equation for Thin Plate	23
4.2 Variational Principle for Thin Plate	27

5 Finite Element Methods for Thin Plate	29
5.1 Non-conforming Finite Element Method	29
5.2 Hybrid-stress Finite Element Method	31
6 Weak-Kirchhoff Plate Element	34
6.1 Theory of Mindlin-Reissner Middle-thickness Plate	34
6.2 Weak-Kirchhoff Condition	37
6.3 $18 - \beta$ Weak-Kirchhoff Triangular (WKT-18) Finite Element	40
6.4 Computational Result	51
7 Theoretical Analysis and Least Order WKT-9 Finite	52
7.1 Discrete BB-condition for Weak-Kirchhoff Finite Element Method	52
7.2 Least order WKT-9 Finite Element	56
8 Conclusion	59
Bibliography	60
Appendix I	61
Program for Solving Laplace Equation with Triangular Element	61
Program for Solving Laplace Equation with Bi-linear Quadratic Element	63
Program for Solving Plate Equation with Weak-Kirchhoff Element.	65
CURRICULUM VITAE	69