

DOCTORAL THESIS

Distance-two constrained labeling and list-labeling of some graphs

Zhou, Haiying

Date of Award:
2013

[Link to publication](#)

General rights

Copyright and intellectual property rights for the publications made accessible in HKBU Scholars are retained by the authors and/or other copyright owners. In addition to the restrictions prescribed by the Copyright Ordinance of Hong Kong, all users and readers must also observe the following terms of use:

- Users may download and print one copy of any publication from HKBU Scholars for the purpose of private study or research
- Users cannot further distribute the material or use it for any profit-making activity or commercial gain
- To share publications in HKBU Scholars with others, users are welcome to freely distribute the permanent URL assigned to the publication

Abstract

The distance-two constrained labeling of graphs arises in the context of frequency assignment problem (FAP) in mobile and wireless networks. The frequency assignment problem is the problem of assigning frequencies to the stations of a network, so that interference between nearby stations is avoided or minimized while the frequency reusability is exploited. It was first formulated as a graph coloring problem by Hale, who introduced the notion of the T-coloring of a graph, and that attracts a lot of interest in graph coloring. In 1988, Roberts proposed a variation of the channel assignment problem in which “close” transmitters must receive different channels and “very close” transmitters must receive channels at least two apart. Motivated by this variation, Griggs and Yeh first proposed and studied the $L(2, 1)$ -labeling of a simple graph with a condition at distance two. Because of practical and theoretical applications, the interest for distance-two constrained labeling of graphs is increasing. Since then, many aspects of the problem and related problems remain to be further explored.

In this thesis, we first give an upper bound of the $L(2, 1)$ -labeling number, or simply λ number, for a special class of graphs, the n -cubes Q_n , where $n = 2^k - k - 1$. Chang et al. [3] considered a generalization of $L(2, 1)$ -labeling, namely, $L(d, 1)$ -labeling of graphs. We study the $L(1, 1)$ -labeling number of Q_n . A lower bound on $\lambda_1(Q_n)$ is provided and $\lambda_1(Q_{2^k-1})$ is determined.

As a related problem, the $L(2, 1)$ -choosability of graphs is studied. Vizing [17] and Erdős et al. [18] generalized the graph coloring problem and introduced the list coloring problem independently more than three decades ago. We shall consider a new variation of the $L(2, 1)$ -labeling problem, the list- $L(2, 1)$ -labeling problem. We determine the $L(2, 1)$ -choice numbers for paths and cycles. We also study the $L(2, 1)$ -choosability for some special graphs such as the Cartesian product graphs and the generalized Petersen graphs. We provide upper bounds of the $L(2, 1)$ -choice numbers for the Cartesian product of a path and a spider, also for the generalized Petersen graphs.

Keywords: distance-two labeling, λ -number, $L(2, 1)$ -labeling, $L(d, 1)$ -labeling, list- $L(2, 1)$ -labeling, choosability, $L(2, 1)$ -choice number, path, cycle, n -cube, spider, Cartesian product graph, generalized Petersen graph.

Table of Contents

Declaration	i
Abstract	ii
Acknowledgements	iii
Table of Contents	iv
List of Figures	vi
Chapter 1 Introduction	1
1.1 Graphs	1
1.2 Frequency assignment problem	2
1.3 Graph labeling problems	4
1.4 Outline of the thesis	9
Chapter 2 $L(1, 1)$ and $L(2, 1)$-labelings for n-cubes	11
2.1 $L(1, 1)$ -labeling for Q_{2^k-1}	12
2.2 Lower bound on $\lambda_1(Q_n)$	16
2.3 $L(2, 1)$ -labeling for Q_{2^k-k-1}	20
Chapter 3 $L(2, 1)$-choosability of paths and cycles	25
3.1 The $L(2, 1)$ -choosability of paths	26
3.2 The $L(2, 1)$ -choosability of cycles	42

Chapter 4	$L(2, 1)$-choosability of Cartesian product of paths and spiders	52
4.1	The $L(2, 1)$ -choosability of Cartesian product of a path and a spider .	53
Chapter 5	$L(2, 1)$-choosability of generalized Petersen graphs	68
5.1	The $L(2, 1)$ -choosability of permutation Petersen graphs	69
Chapter 6	Conclusion and future works	70
6.1	Summary of the thesis	70
6.2	Further research	71
Curriculum Vitae		83