DOCTORAL THESIS

Geometric transformation and image singularity with wavelet analysis

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Geometric Transformation and Image Singularity with Wavelet Analysis

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Abstract

Since later 1980s, wavelet analysis has been a popular field in scientific research. Wavelet analysis has been developed relatively fast and its applications have been extended widely in many scientific fields, such as image processing, signal processing and so on, and become such a voluminous subjects in recent years. In this thesis, we are mainly concerned on the analysis of geometric transformation and image singularity with wavelet theory. This thesis is organized in two major parts, and has four main contributions. The first part consist of three main contributions and the second part possesses one.

The first part of the thesis deals with the research of geometric transformation using wavelet theory. This part consists of three main contributions, which are presented briefly below:

Dyadic and decimal wavelet transforms have been used to derive affine invariant functions. The invariant functions are based on the dyadic wavelet transform as well as decimal one of the object boundary. The boundary-based approach for processing the affine transform is reviewed, and some experimental comparisons and evaluations are provided in this thesis. Six invariant functions have been calculated using different numbers of dyadic and decimal levels. Experimental results show that the stability of these invariant functions have been successfully tested for a great number of transformations.

To improve the boundary-based approach, a region-based approach is then developed in this thesis. According to theoretical analysis, the shape transformation model can be regarded as a generation-model, and its inverse transformation can be considered to be an invariant function or normalization-model. The generation-model and normalization-model are amenable to the generation of shapes and normalization of shapes respectively. Elastic geometric transformation model, i. e. harmonic model, can transform an image into arbitrary shapes including the affine one. Harmonic model can be represented by a partial differential equation with boundary condition.

Typical numerical methods, which are available for partial differential equation such as finite element method, finite difference method and moment method are discussed. Based on the discussion, a novel approach based on wavelet analysis is proposed to handle the harmonic transformation, which is called Integral Equation-Wavelet Collocation (IEWC) method. In this approach, first, an indirect method is used so that the partial differential equation (PDE) (Laplace's equation) is changed into the form of integral equation and integral representation on boundary, which are called boundary integral equation (BIE) and boundary integral representation (BIR) respectively. In order to solve the boundary integral equation (BIE) efficiently and to get rid of the second defect in the finite element method, the boundary measure formula (BMF) is used. It changes the boundary integral equation and boundary integral representation into an integral equation and an integral representation on the whole plane R^2 rather than the special boundary. They are called plane integral equation (PIE) and plane integral representation (PIR) respectively. In this way, when boundary is changed, the program code will not to be modified at all. Then, wavelet collocation technique is used to solve the plane integral equation. In the integral equation, the integrand has a discontinuity across boundary. Hence, there is a kind of singularity in it. Fortunately, wavelets have a good property to approximate this kind of singularity. Therefore, the solution of the harmonic transformation can be found. This novel approach is the most significant contribution of this thesis.

Another novel approach to handle partial differential equation for harmonic transformation is presented in this thesis, which consists of three stages: (1) Laplace equation (LE) is converted into a boundary integral equation (BIE) by the potential theory. (2) Based on the moment method, the boundary integral equation (BIE) is changed to a dense matrix equation (DME). Using the moment method, boundary is divided into several smooth pieces. The middle point of each piece of boundary, which equals to use Kronecker delta function as the weighting function, a matrix equation can then be derived. (3) The dense matrix equation (DME) is transformed to a sparse matrix equation (SME) according to the wavelet matrix transform. It can considerably reduce the computational cost. We give two ways to construct a wavelet matrix, which are based on scalar wavelet and multi-wavelet. Almost all the scalar wavelets, either othogonal or bi-orthogonal ones, can be used to produce wavelet matrix in this method. However, many problems may occur, when multi-wavelet is used. In this thesis, a preprocessing called spatial balance is utilized to solve these problems.

The second part of the thesis deals with the research of image singularity with wavelet analysis. One main contribution is involved in this part.

A modified algorithm for corner detection using wavelet transform is also developed in this thesis. The modified algorithm depends on a special wavelet function, which is designed specifically for corner features. This method can utilize both the information of the local extrema and modulus of transform results to detect corners and arcs effectively.

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