DOCTORAL THESIS

Some operator splitting methods for convex optimization

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Abstract

Many applications arising in various areas can be well modeled as convex optimization models with separable objective functions and linear coupling constraints. Such areas include signal processing, image processing, statistical learning, wireless networks, etc. If these well-structured convex models are treated as generic models and their separable structures are ignored in algorithmic design, then it is hard to effectively exploit the favorable properties that the objective functions possibly have. Therefore, some operator splitting methods have regained much attention from different areas for solving convex optimization models with separable structures in different contexts.

In this thesis, some new operator splitting methods are proposed for convex optimization models with separable structures. We first propose combining the alternating direction method of multiplier with the logarithmic-quadratic proximal regularization for a separable monotone variational inequality with positive orthant constraints and propose a new operator splitting method. Then, we propose a proximal version of the strictly contractive Peaceman-Rachford splitting method, which was recently proposed for the convex minimization model with linear constraints and an objective function in form of the sum of two functions without coupled variables. After that, an operator splitting method suitable for parallel computation is proposed for a convex model whose objective function is the sum of three functions. For the new algorithms, we establish their convergence and estimate their convergence rates measured by the iteration complexity. We also apply the new algorithms to solve some applications arising in the image processing area; and report some preliminary numerical results. Last, we will discuss a particular video processing application and propose a series of new models for background extraction in different scenarios; to which some of the new methods are applicable.

Keywords: Convex optimization, Operator splitting method, Alternating direction method of multipliers, Peaceman-Rachford splitting method, Image processing

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