

## DOCTORAL THESIS

### Some operator splitting methods for convex optimization

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# Abstract

Many applications arising in various areas can be well modeled as convex optimization models with separable objective functions and linear coupling constraints. Such areas include signal processing, image processing, statistical learning, wireless networks, etc. If these well-structured convex models are treated as generic models and their separable structures are ignored in algorithmic design, then it is hard to effectively exploit the favorable properties that the objective functions possibly have. Therefore, some operator splitting methods have regained much attention from different areas for solving convex optimization models with separable structures in different contexts.

In this thesis, some new operator splitting methods are proposed for convex optimization models with separable structures. We first propose combining the alternating direction method of multiplier with the logarithmic-quadratic proximal regularization for a separable monotone variational inequality with positive orthant constraints and propose a new operator splitting method. Then, we propose a proximal version of the strictly contractive Peaceman-Rachford splitting method, which was recently proposed for the convex minimization model with linear constraints and an objective function in form of the sum of two functions without coupled variables. After that, an operator splitting method suitable for parallel computation is proposed for a convex model whose objective function is the sum of three functions. For the new algorithms, we establish their convergence and estimate their convergence rates measured by the iteration complexity. We also apply the new algorithms to solve some applications arising in the image processing area; and report some preliminary numerical results. Last, we will discuss a particular video processing application and propose a series of new models for background extraction in different scenarios; to which some of the new methods are applicable.

**Keywords:** Convex optimization, Operator splitting method, Alternating direction method of multipliers, Peaceman-Rachford splitting method, Image processing

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# Contents

<b>Abstract</b>	<b>ii</b>
<b>Acknowledgements</b>	<b>iii</b>
<b>Table of Contents</b>	<b>v</b>
<b>List of Figures</b>	<b>ix</b>
<b>List of Tables</b>	<b>xii</b>
<b>Chapter 1 Introduction</b>	<b>1</b>
1.1 Some convex programming models . . . . .	1
1.2 Notations . . . . .	2
1.3 Convex model (P1) . . . . .	5
1.3.1 Optimality condition for (P1) . . . . .	5
1.3.2 Augmented Lagrangian method . . . . .	6
1.3.3 A typical application: basis pursuit . . . . .	6
1.4 Convex model (P2) . . . . .	7
1.4.1 Optimality condition for (P2) . . . . .	7
1.4.2 Some operator splitting methods for solving (P2) . . . . .	8
1.4.2.1 The alternating direction method of multipliers (ADMM) . . . . .	8
1.4.2.2 Generalized ADMM . . . . .	10
1.4.2.3 Linearized ADMM . . . . .	10
1.4.2.4 The Peaceman-Rachford splitting method (PRSM) . . . . .	12
1.4.2.5 The strictly contractive PRSM . . . . .	13

1.4.3	A typical application: image restoration . . . . .	14
1.5	A variational inequality with positive orthants (VI+) . . . . .	15
1.5.1	The ADMM for VI+ . . . . .	15
1.5.2	Generalized ADMM for VI+ . . . . .	16
1.5.3	Quadratic-proximal regularized ADMM for VI+ . . . . .	16
1.5.4	LQP-regularized ADMM for VI+ . . . . .	17
1.6	Convex model (P3) . . . . .	18
1.6.1	Optimality condition for (P3) . . . . .	18
1.6.2	Some operator splitting methods for solving (P3) . . . . .	19
1.6.2.1	The direct application of ADMM . . . . .	19
1.6.3	A typical application: image decomposition . . . . .	21
1.7	Organization of the thesis . . . . .	22
 <b>Chapter 2 A generalized ADMM with LQP regularization for a class of variational inequalities</b>		<b>24</b>
2.1	Algorithm . . . . .	25
2.2	Global convergence . . . . .	25
2.3	Convergence rate . . . . .	33
2.3.1	Convergence rate in an ergodic sense . . . . .	33
2.3.2	Convergence rate in a nonergodic sense . . . . .	34
 <b>Chapter 3 A proximal strictly contractive PRSM for (P2)</b>		<b>36</b>
3.1	Algorithm . . . . .	36
3.2	Global convergence . . . . .	38
3.3	Convergence rate . . . . .	48
3.3.1	Convergence rate in an ergodic sense . . . . .	48
3.3.2	Convergence rate in a nonergodic sense . . . . .	49
3.4	Applications to image processing . . . . .	51
3.4.1	A wavelet-based inpainting model . . . . .	52
3.4.1.1	Application background . . . . .	53
3.4.1.2	Implementation of PSC-PRSM . . . . .	53

3.4.1.3	Comparison with SC-PRSM . . . . .	55
3.4.1.4	Comparison with other benchmarks . . . . .	58
3.4.2	The computational tomography problem . . . . .	63
3.4.2.1	Application background . . . . .	63
3.4.2.2	The implementation of PSC-PRSM . . . . .	64
3.4.2.3	Comparison with SC-PRSM . . . . .	66
3.4.2.4	Comparison with ADMM . . . . .	68
<b>Chapter 4</b>	<b>A parallel operator splitting algorithm for (P3)</b>	<b>74</b>
4.1	Algorithm . . . . .	74
4.3	Global convergence . . . . .	78
4.4	Application to Retinex . . . . .	82
4.4.1	A new model . . . . .	83
4.4.2	Numerical implementation . . . . .	86
4.4.3	Numerical results . . . . .	88
<b>Chapter 5</b>	<b>Median filter based variational models for background ex-</b>	
	<b>traction</b>	<b>99</b>
5.1	Background and motivation . . . . .	100
5.2	Numerical implementation . . . . .	104
5.3	Numerical results . . . . .	108
5.3.1	Comparison between MED and RPCA . . . . .	110
5.3.2	Comparison between MED-i and RPCA-i . . . . .	111
5.3.3	Comparison between MED-ii and RPCA-ii . . . . .	118
5.4	Concluding Remarks . . . . .	123
<b>Chapter 6</b>	<b>Conclusions and future work</b>	<b>125</b>
6.1	Conclusions . . . . .	125
6.1.1	Algorithm Proposition . . . . .	125
6.1.2	Convergence Analysis . . . . .	126
6.1.3	Applications . . . . .	126

6.2 Future work . . . . .	126
<b>References</b>	<b>128</b>
<b>Curriculum Vitae</b>	<b>144</b>