

DOCTORAL THESIS

First-order affine scaling continuous method for convex quadratic programming

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Abstract

We develop several continuous method models for convex quadratic programming (CQP) problems with different types of constraints. The essence of the continuous method is to construct one ordinary differential equation (ODE) system such that its limiting equilibrium point corresponds to an optimal solution of the underlying optimization problem. All our continuous method models share the main feature of the interior point methods, i.e., starting from any interior point, all the solution trajectories remain in the interior of the feasible regions.

First, we present an affine scaling continuous method model for nonnegativity constrained CQP. Under the boundedness assumption of the optimal set, a thorough study on the properties of the ordinary differential equation is provided, strong convergence of the continuous trajectory of the ODE system is proved. Following the features of this ODE system, a new ODE system for solving box constrained CQP is also presented. Without projection, the whole trajectory will stay inside the box region, and it will converge to an optimal solution. Preliminary simulation results illustrate that our continuous method models are very encouraging in obtaining the optimal solutions of the underlying optimization problems.

For CQP in the standard form, the convergence of the iterative first-order affine scaling algorithm is still open. Under boundedness assumption of the optimal set and nondegeneracy assumption of the constrained region, we discuss the properties of the ODE system induced by the first-order affine scaling direction. The strong convergence of the continuous trajectory of the ODE system is also proved.

Finally, a simple iterative scheme induced from our ODE is presented for finding an optimal solution of nonnegativity constrained CQP. The numerical results illustrate the good performance of our continuous method model with this iterative scheme.

Keywords: ODE; Continuous method; Quadratic programming; Interior point method; Affine scaling.

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Table of Contents

Declaration	i
Abstract	ii
Acknowledgements	iii
Table of Contents	iv
List of Tables	vi
List of Figures	vii
Chapter 1 Introduction	1
1.1 Framework of the Continuous Method	2
1.2 Preliminaries	3
1.2.1 Ordinary Differential Equation	3
1.2.2 Convex Programming	5
1.2.3 Notation	6
1.3 Outline of the Thesis	6
Chapter 2 First-order Affine Scaling Continuous Method for Nonnegativity and Box Constrained CQP	8
2.1 Introduction	8
2.2 An Affine Scaling Continuous Method for Nonnegativity Constrained CQP	11
2.2.1 Construction of an ODE	11
2.2.2 Properties for the Continuous Trajectory	14
2.2.3 Strong Convergence	21

2.3	An Affine Scaling Continuous Method for Box Constrained CQP	27
2.4	Numerical Illustration	39
2.4.1	The Performance of the Continuous Method for Nonnegativity Constrained CQP	40
2.4.2	The Performance of the Continuous Method for Box Constrained CQP	42
2.5	Concluding Remarks	44
Chapter 3 First-order Affine Scaling Continuous Trajectory for Stan-		
	 dard CQP	46
3.1	Introduction	46
3.2	Central Path and Affine Scaling Direction	49
3.3	Properties of the Continuous Trajectory	51
3.4	Optimality of Cluster Point	55
3.5	Strong Convergence	59
3.6	Concluding Remarks	68
Chapter 4 Summary		69
4.1	Summary of the Thesis	69
4.2	Future Research	70
Bibliography		74
Curriculum Vitae		84