Continuous methods for convex programming and convex semidefinite programming
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Abstract

In this thesis, we study several interior point continuous trajectories for linearly constrained convex programming (CP) and convex semidefinite programming (SDP). The continuous trajectories are characterized as the solution trajectories of corresponding ordinary differential equation (ODE) systems. All our ODE systems are closely related to interior point methods.

First, we propose and analyze three continuous trajectories, which are the solutions of three ODE systems for linearly constrained convex programming. The three ODE systems are formulated based on an variant of the affine scaling direction, the central path, and the affine scaling direction in interior point methods. The resulting solutions of the first two ODE systems are called generalized affine scaling trajectory and generalized central path, respectively. Under some mild conditions, the properties of the continuous trajectories, the optimality and convergence of the continuous trajectories are all obtained. Furthermore, we show that for the example of Gilbert et al. [Math. Program., 103, 63-94 (2005)], where the central path does not converge, our generalized central path converges to an optimal solution of the same example in the limit.

Then we analyze two primal dual continuous trajectories for convex programming. The two continuous trajectories are derived from the primal-dual path-following method and the primal-dual affine scaling method, respectively. Theoretical properties of the two interior point continuous trajectories are fully studied. The optimality and convergence of both interior point continuous trajectories are obtained for any interior feasible point under some mild conditions. In particular, with proper choice of some parameters, the convergence for both continuous trajectories does not require the strict complementarity or the analyticity of the objective function.

For convex semidefinite programming, four interior continuous trajectories defined by matrix differential equations are proposed and analyzed. Optimality and convergence of the continuous trajectories are also obtained under some mild conditions.
We also propose a strategy to guarantee the optimality of the affine scaling algorithm for convex SDP.

**Keywords:** Ordinary differential equation; Interior point method; Continuous trajectory; Affine scaling; Convex programming; Convex semidefinite programming.
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