

## DOCTORAL THESIS

### The common self-polar triangle of conics and its applications to computer vision

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# Abstract

In projective geometry, the common self-polar triangle has often been used to discuss the location relationship of two planar conics. However, there are few researches on the properties of the common self-polar triangle, especially when the two planar conics are special conics. In this thesis, the properties of the common self-polar triangle of special conics are studied and their applications to computer vision are presented. Specifically, the applications focus on the two topics of the computer vision: camera calibration and homography estimation.

This thesis first studies the common self-polar triangle of two sphere images and also that the common self-polar triangle of two concentric circles, and exploits its properties to solve the problem of camera calibration. For the sphere images, by recovering the constraints on the imaged absolute conic from the vertices of the common self-polar triangles, a novel method for estimating the intrinsic parameters of a camera from an image of three spheres has been developed. For the other case of concentric circles, it is shown in this thesis that the imaged circle center and the vanishing line of the support plane can be recovered simultaneously. Furthermore, many orthogonal vanishing points can be obtained from the common self-polar triangles. Consequently, two novel calibration methods have been developed. Based on our method, one of the state-of-the-art calibration methods has been well interpreted.

This thesis then studies the common self-polar triangle of two separate ellipses, and applies it to planar homography estimation. For two images of the separate ellipses, by inducing four corresponding lines from the common self-polar triangle,

a homography estimation method has been developed without ambiguity. Based on these results, a special case of planar rectification with two identical circles is also studied. It is shown that given one image of the two identical circles, the vanishing line of the support plane can be recovered from the common self-polar triangle and the imaged circle points can be obtained by intersecting the vanishing line with the image of the circle. Accordingly, a novel method for estimating the rectification homography has been developed and experimental results show the feasibility of our method.

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