MASTER'S THESIS

Numerical methods for solving Markov chain driven Black-Scholes model

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Numerical Methods for Solving Markov Chain Driven Black-Scholes Model

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Abstract

Option pricing plays an important role in the financial market nowadays, and therefore, different option pricing models are developed. One of the famous option pricing models is the Black-Scholes model (BSM) which is widely used because of its good adaptive property for different types of options. The BSM develops a partial differential equation (PDE) that the solution is the option value. Solving the BSM-PDE exactly is not easy or even impossible. Instead, we propose to use the method of fundamental solutions (MFS) to evaluate the solution numerically.

To derive the BSM with the consideration of easy implementation, the risk-free interest rate r and the volatility σ are simply assumed to keep constants along the whole lifespan of the option. Since the risk-free interest rate and the volatility change due to the changes of the economy status, these assumptions are not realistic in the market if the lifespan of the option is long. To tackle the problem of changeable parameters, we apply the backward-Markov-regime-switching to model the dynamic of the Markov-driven risk-free interest rate $r_{\mathcal{M}}(t)$ and the Markov-driven volatility $\sigma_{\mathcal{M}}(t)$ at time t. Thus, we derive a modified BSM—Backward Markov Black-Scholes model (BMBSM) and we use the MFS as our numerical solver to evaluate the option price with the consideration of the economy status.

In this thesis, we numerically evaluate the solution of the BMBSM-PDE for European options and American options. Detail discussion about the adaptive of the BMBSM, the implementation of our numerical procedure and the financial meaning of the numerical results are also provided.

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