

DOCTORAL THESIS

Wavelet and manifold learning and their applications

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Wavelet and Manifold Learning and Their Applications

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for the degree of
Doctor of Philosophy

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Abstract

The wavelet transform is viewed as a synthesis of ideas from different fields, such as mathematics and signal processing. Generally, the wavelet transform is a tool that divides up data, functions, or operators into different frequency components and then studies each component with a resolution matched to its scale. Therefore, the wavelet transform is anticipated to provide economical and informative mathematical representation of many objects of interest. Wavelets are applied to a diverse set of problems and have made significant technology advancement in computer science research areas such as image processing, computer vision, network management, and data mining. Wavelets enjoy various properties including vanishing moments, hierarchical and multiresolution decomposition structure, linear time and space complexity of the transformations, decorrelated coefficients, and a wide variety of basis functions. These properties could provide considerably more efficient and effective solutions to solve many problems. In this thesis, we apply wavelet analysis to various fields and obtain the good performance.

Manifold learning is a popular recent approach to nonlinear dimensionality reduction. With the development of science and technology, more and more signals need to be processed, especially for high dimensional data. Modern data sets often consist of a large number of examples, each of which is made up of many features. Thus many problems in information processing involve some form of dimensionality reduction. Generally speaking, this input data which can lead to “curse of dimensionality” is suspected to be notoriously redundant. Obviously, “redundant” means that there is much data but less information in the input set. Furthermore, each data point may be described as a function of only a few underlying parameters although it consists of perhaps thousands of features. That is, the data points are actually samples from a low-dimensional manifold which is embedded in a high-dimensional space. Manifold learning algorithms attempt to uncover these parameters in order to find a low-dimensional representation of the data. It is necessary to perform the desired task using extracted features instead of the raw input data. This thesis studies the approaches to dimensionality reduction based on Tensor Locality Preserving

Projections and Gabor-based tensor local discriminant embedding.

The contents of this thesis involve primarily the following aspects:

- Face recognition is one of the few biometric methods that possess the merits of both high accuracy and low intrusiveness. It has the accuracy of a physiological approach without being intrusive. For this reason, since the early 70's, face recognition has drawn the attention of researchers in the fields from security, psychology, image processing, and computer vision. Numerous algorithms have been proposed for face recognition. Feature extraction is very important because effectiveness of identification system depends on it. There are many different methods to extract features such as eigenface, Fourier transform etc. Using wavelet transform and wavelet packet transform, we extract the features of face imaged and design the face recognition systems.
- Textures are repetitive visual patterns for surfaces of objects and provide important information for object segmentation and identification. Segmentation of a texture image is a difficult task. Many researchers suggest that wavelet analysis is a good method because it can provide the joint space/frequency resolution. We propose a novel approach to frequency segmentation using wavelet decomposition of *pseudo-motion image*. In this way, a fixed image is translated such that a sequence of moving images is produced, which are called the *pseudo-motion images*. In fact, we can consider the translation of a function to be a motion of eyeshot. When a function is translated, its wavelet coefficients will oscillate. From this property, we can detect the special areas of the image.
- In recent years, the BEM (boundary element method) has been widely used in the computation. The most attractive feature of the boundary element for the solution of many problems is to reduce dimension. The discrete compactly supported wavelets have many good features such as orthogonality, compactly supported. So the wavelet-Galerkin methods have been applied to get the numerical solutions of partial differential equations. To solve PDE, we need study the integral equations in depth. The singularity is the difficult problem for integral equations. In this thesis, Wavelet-Galerkin algorithm for solving

the first kind of weak singular integral equations with the logarithmic kernel is proposed.

- Recently, to solve the problem dimensionality reduction, some new linear manifold learning techniques have been proposed. We introduce Locality Preserving Projections (LPP), Tensor LPP (TLPP). LPP should be seen as an alternative to Principal Component Analysis (PCA) - a classical linear technique that projects the data along the directions of maximal variance. When the high dimensional data lies on a low dimensional manifold embedded in the ambient space, the LPP is obtained by finding the optimal linear approximations to the eigenfunctions of the Laplace Beltrami operator on the manifold. As a result, LPP shares many of the data representation properties of nonlinear techniques such as Laplacian Eigenmaps or Locally Linear Embedding. But LPP only processes vectorized data, it also suffer from the curse of dimensionality and the high computation. In the thesis, a new multilinear approach to face recognition - Tensor Locality Preserving Projections (TLPP) algorithm is proposed. TLPP is a natural extension of LPP to the multilinear case. It is particularly useful in applications when the data samples are naturally represented as matrices or higher-order tensors.
- To reduce dimensionality, we attempt to apply Tensor Local Discriminant Embedding (TLDE) and Gabor-based TLDE (GTLDE). In this thesis, a novel feature extraction method called Gabor-based tensor local discriminant embedding (GTLDE) is proposed. GTLDE firstly gets the high-order statistic information by using a biologically inspired hierarchical model, and then tensor local discriminant embedding (TLDE) is carried out to extract the discriminant features of the image for recognition task. The method we proposed is not only robust to local translation and scale variations, but also has high distinguishing ability. GTLDE are presented to reduce dimension in experiments.

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