

## DOCTORAL THESIS

### Conservative and non-conservative optical forces

Li, Xiao

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## **Abstract**

The fact that optical force is very significant in the microscopic world and can be used to manipulate microparticles has triggered an evolution in micromanipulation, in particular, the manipulation of biological species and colloidal particles. The induced optical force can easily be more than  $10^3$  times of the particle's weight. The particle size that are accessible to optical forces ranges from tens of nanometers to hundreds of micrometers. One of the most well-known tools in optical manipulation is called optical tweezers, which is, in essence, performing optical trapping by a strongly focused light beam. The optical force induced by the incident light wave can be generally decomposed into two mathematically and physically distinct components, namely the conservative (gradient force) and non-conservative (scattering and absorption force) forces. Such a split helps in the study of optical forces and elucidates the underlying physics (e.g., the optical trapping). For example, in optical trapping, the conservative gradient force drives the particles toward the intensity maxima and traps the particles there, whereas the non-conservative scattering and absorption force tends to push the particles away and thus has some destabilizing effects. However, while a significant portion of paper dealing with optical trapping explicitly mentioned gradient and scattering forces, the true and exact force profiles of the decomposed optical forces have been mysteries for decades. Researchers still use these concepts, and to certain extent, they imagine the force profile according to their own convenience. This thesis is mainly devoted to the analytical and numerical studies of the decomposition of optical forces. The intrinsic nature of the decomposed optical forces will be discussed, and the approaches of generating a purely conservative force field are presented.

First, the analytical approaches for decomposing the optical force into the gradient force and the scattering and absorption force are described. These approaches can be applied to different particle sizes (smaller than 40% of the wavelength if the multipoles are only considered up to the electric octopole or much larger than the wavelength under the geometrical optics limit), but they still cannot describe the experimentally accessible particle size, which is on the order of micrometer. Second, within the dipole limit, the origin of scattering force is shown to be resulted from the radiation reaction, the polarizations, and the topological charges. In addition, it is found that the conservativeness of the force is closely related to the force constant matrix (the linear term in the Taylor expansion of the optical force) at every point, and certain symmetries in these force constant matrix can guarantee the force to be conservative.

A numerical method that utilizes the fast Fourier transform (FFT) was developed to decompose the conservative and non-conservative forces. This approach is valid when the total force field is spatially localized and decayed sufficiently fast as we move away from the beam center (e.g., optical tweezers or alike) or is spatially periodic (e.g. plane incident waves).

We also considered spherical aberration due to the mismatch of the refractive indices between the oil and water media in a typical optical tweezers setup within the FFT method. Various particle sizes, materials, and numerical apertures were also considered. For the periodic force field generated by a collection of plane waves, it is demonstrated that an incident 2-dimensional standing wave could generate a purely conservative force field. The accuracy of this fast Fourier transform approach is analyzed in details and shown to be quite accurate. Moreover, an incident 3-dimensional standing wave could also induce a conservative force field for intermediately sized particles.

Finally, three counter-intuitive examples obtained with the fast Fourier transform approach are presented. These examples clearly demonstrated the need to calculate the gradient and scattering forces accurately, as not doing so would lead to qualitatively wrong results.

# Table of Contents

<b>Declaration</b> .....	i
<b>Abstract</b> .....	ii
<b>Acknowledgement</b> .....	v
<b>Table of Contents</b> .....	vii
<b>List of Figures</b> .....	xi
<b>Chapter 1: Introduction</b> .....	1
1.1 The Importance of Optical Forces.....	1
1.2 The Historical Landmarks in Optical Micromanipulation.....	2
1.3 Introduction to the Gradient Force and the Scattering Force.....	6
1.4 Organization of This Thesis.....	10
<b>Chapter 2: Analytical Theory</b> .....	14
2.1 Conservative and Non-Conservative Forces Acting on Rayleigh Particles.....	15
2.1.1 <i>Time-averaged Optical Forces Exerted on a Single Neutral Rayleigh Particle</i> .....	15
2.1.2 <i>Analytical Expressions of Conservative and Non-Conservative Forces for a Rayleigh Particle</i> .....	19
2.2 Multipole Expansion of Conservative and Non-Conservative Optical Forces of Moderately Sized Particles.....	24
2.2.1 <i>Optical Force for a Moderately Sized Particle and Its Expression in Multipole Expansion</i> .....	24

2.2.2 <i>Conservative and Non-Conservative Optical Forces with Multipole Expansion up to Electric Octopole</i> .....	30
2.3 Conservative and Non-Conservative Optical Forces in Geometrical Optics Regime .....	36
2.4 Origin and Mechanism of Optical Forces .....	41
2.4.1 <i>Origin of Non-Conservative Scattering Forces Acting on Rayleigh Particles due to Radiative Reaction</i> .....	41
2.4.2 <i>Non-Conservative Scattering Forces and the Topological Charges and Polarizations</i> .....	44
2.4.3 <i>Sufficient Conditions to Generate a Conservative Force Field</i> .....	47
<b>Chapter 3: Numerical Methods for Calculating the Incident Fields and Scattered Fields</b> .....	53
3.1 Incident Fields with Aberration.....	54
3.1.1 <i>Derivation of the Incident Fields with Aberration</i> .....	54
3.1.2 <i>Numerical Simulation to Show the Incident Fields with Aberration</i> .....	63
3.2 Incident Fields without Aberration .....	65
3.3 Mie Theory to Solve the Scattered Electromagnetic Fields by Particles .....	68
3.3.1 <i>Scattered Fields of a Spherical Particle</i> .....	69
3.3.2 <i>Scattered Fields of an Infinitely Long Cylinder</i> .....	71
3.4 Forces Exerted on an Infinitely Long Cylinder and the Validity of This 2-Dimensional Model .....	74
<b>Chapter 4: Gradient Force and Scattering Force Calculated by Fast Fourier Transform</b> .....	79

4.1 Illustration of the FFT Method.....	81
4.2 Calculation of Gradient Force and Scattering Force for the Localized Force Field in Optical Tweezers .....	82
4.2.1 <i>An illustrative Example</i> .....	82
4.2.2 <i>Gradient Force and Scattering Force for Particles with Different Sizes and Materials.....</i>	85
4.2.3 <i>Numerical Aperture</i> .....	94
4.2.4 <i>Aberration.....</i>	97
4.2.5 <i>On the Requirement of a Sufficiently Fast Decaying Field</i> .....	99
4.2.6 <i>On the Accuracy for the FFT Method</i> .....	101
4.2.7 <i>Comparison between the Numerical and Analytical Results</i> .....	108
4.3 Calculation of Gradient Force and Scattering Force for Spatially Periodic Force Pattern Generated by Plane Waves .....	109
4.3.1 <i>Gradient Force and Scattering Force Generated by Plane Waves</i> .....	109
4.3.2 <i>Conservativeness of the Periodic Force Field Generated by TE or TM Standing Waves in 2-Dimensions and in 3-Dimensions.....</i>	111
<b>Chapter 5: Counter-Intuitive Results Obtained Using the Fast Fourier Transform Method.....</b>	<b>120</b>
5.1 Scattering Force Determines the Stability of a Particle in Optical Tweezers ..	120
5.2 Potential Energy Profiles in Optical Tweezers Changing with the Dielectric Constants .....	122
5.3 Generation of a Conservative Force Field with Three Plane Waves in 2- Dimensions.....	125

<b>Chapter 6: Conclusion and Discussion .....</b>	<b>127</b>
<b>Appendices.....</b>	<b>130</b>
<b>Appendix 1: Incident Fields Plots for Each Component of Electric and Magnetic Fields .....</b>	<b>130</b>
<b>Appendix 2: Force Derivation Procedure of an Infinitely Long Cylinder .....</b>	<b>134</b>
<b>Appendix 3: Torque Derivation Procedure of an Infinitely Long Cylinder and Its Invalidity .....</b>	<b>140</b>
<b>Appendix 4: Useful Expressions for Fields Related Terms in Appendix 2 and Appendix 3 .....</b>	<b>163</b>
<b>Appendix 5: Supplementary Figures.....</b>	<b>172</b>
<b>List of References.....</b>	<b>173</b>
<b>Curriculum Vitae .....</b>	<b>182</b>