

DOCTORAL THESIS

Conservative and non-conservative optical forces

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Abstract

The fact that optical force is very significant in the microscopic world and can be used to manipulate microparticles has triggered an evolution in micromanipulation, in particular, the manipulation of biological species and colloidal particles. The induced optical force can easily be more than 10^3 times of the particle's weight. The particle size that are accessible to optical forces ranges from tens of nanometers to hundreds of micrometers. One of the most well-known tools in optical manipulation is called optical tweezers, which is, in essence, performing optical trapping by a strongly focused light beam. The optical force induced by the incident light wave can be generally decomposed into two mathematically and physically distinct components, namely the conservative (gradient force) and non-conservative (scattering and absorption force) forces. Such a split helps in the study of optical forces and elucidates the underlying physics (e.g., the optical trapping). For example, in optical trapping, the conservative gradient force drives the particles toward the intensity maxima and traps the particles there, whereas the non-conservative scattering and absorption force tends to push the particles away and thus has some destabilizing effects. However, while a significant portion of paper dealing with optical trapping explicitly mentioned gradient and scattering forces, the true and exact force profiles of the decomposed optical forces have been mysteries for decades. Researchers still use these concepts, and to certain extent, they imagine the force profile according to their own convenience. This thesis is mainly devoted to the analytical and numerical studies of the decomposition of optical forces. The intrinsic nature of the decomposed optical forces will be discussed, and the approaches of generating a purely conservative force field are presented.

First, the analytical approaches for decomposing the optical force into the gradient force and the scattering and absorption force are described. These approaches can be applied to different particle sizes (smaller than 40% of the wavelength if the multipoles are only considered up to the electric octopole or much larger than the wavelength under the geometrical optics limit), but they still cannot describe the experimentally accessible particle size, which is on the order of micrometer. Second, within the dipole limit, the origin of scattering force is shown to be resulted from the radiation reaction, the polarizations, and the topological charges. In addition, it is found that the conservativeness of the force is closely related to the force constant matrix (the linear term in the Taylor expansion of the optical force) at every point, and certain symmetries in these force constant matrix can guarantee the force to be conservative.

A numerical method that utilizes the fast Fourier transform (FFT) was developed to decompose the conservative and non-conservative forces. This approach is valid when the total force field is spatially localized and decayed sufficiently fast as we move away from the beam center (e.g., optical tweezers or alike) or is spatially periodic (e.g. plane incident waves).

We also considered spherical aberration due to the mismatch of the refractive indices between the oil and water media in a typical optical tweezers setup within the FFT method. Various particle sizes, materials, and numerical apertures were also considered. For the periodic force field generated by a collection of plane waves, it is demonstrated that an incident 2-dimensional standing wave could generate a purely conservative force field. The accuracy of this fast Fourier transform approach is analyzed in details and shown to be quite accurate. Moreover, an incident 3-dimensional standing wave could also induce a conservative force field for intermediately sized particles.

Finally, three counter-intuitive examples obtained with the fast Fourier transform approach are presented. These examples clearly demonstrated the need to calculate the gradient and scattering forces accurately, as not doing so would lead to qualitatively wrong results.

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