DOCTORAL THESIS

Cycles and coloring in graphs

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Cycles and Coloring in Graphs

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Abstract

In graph theory, cycle is a basic and important concept. The famous century-old hamiltonian and traveling salesman problems are classical examples of problems on cycles. The research on cycles in graphs and digraphs is one of the most active topics.

So far, there are many sufficient conditions for the existence of hamiltonian graphs such as: Ore's condition, Fan's condition, neighborhoods intersection condition and Chvǎtal and Erdős's condition, and so on. Almost all of them only apply to graphs G with large edge density and small diameter. Recent years, some local conditions have been obtained. They apply to infinite classes of graphs G with small edge density and large diameter as well. In Chapter 2, we will give a basic and important result (see Theorem 2.1), which is a local condition. By an idea of "or" and the result, we will prove several results(see Chapters 4 and 5) for hamiltonicity. The conditions of these results are local conditions, which generalize many known results.

Bondy and Chvǎtal define the k(degree)-closure of G, denoted $C_k(G)$. This is a useful method for research of hamiltonicity. The closure concept opened a new horizon for the research on hamiltonian and related properties of graphs. It now play an important role in research on the existence of cycles, paths and other subgraphs in graphs. Inspired by the result several other closure concepts were developed. In Chapter 3, using Theorem 2.1, we define some new closures. Some of them are local conditions.

For graphs, many basic results on sufficient conditions for them to be Hamiltonian also imply that they are pancyclic and/or panconnected. In Chapter 6, by an idea of "or", we give a new sufficient condition for vertex pancyclicity(see Theorem 6.1). We will also study edge pancyclicity of Coupled graphs (Theorem 6.2) induced by a plane graph.

Coloring problems involve partitioning certain specified elements of a graph

into sets, members in each of which are pairwisely non-incidental. The famous four-color problem is one of coloring problems. Although the four-color problem has been changed into the four-color theorem, the research on coloring of graphs is still another active topic in graph theory for several reasons. First, graph coloring theory has a central position in discrete mathematics. It appears in many places with seemingly no or little connection to coloring. Next, graph coloring theory is of interest for its applications. Graph coloring deals with the fundamental problem of partitioning a set of objects into classes, according to certain rules. Time tabling, sequencing, and scheduling problems, in their many forms, are basically of this nature. Third, graph coloring theory continually surprises by producing unexpected new answers.

In the thesis, we discuss the list coloring, list improper coloring and circular chromatic number. Chapter 7 discusses list coloring of plane graphs and gives two results for 4-choosable and 3-choosable. Chapter 8 presents two results about $(3,1)^*$–choosable and $(2,d)^*$–choosable. We also give some examples to show that our results are best possible.

For circular chromatic number, a good survey is given by Zhu. In Chapter 9, we discuss the circular chromatic number of products of graphs. In Chapter 10, we develop a graph transformation which transforms a graph G into a new infinite set of graphs $\{\mu_m(G), m = 0, 1.2, \ldots\}$, we call them Generalized Mycielskians of the graph G. We established many basic properties of $\mu_m(G)$. We also consider the circular chromatic number of $\mu_m(G)$ and prove that for any odd integer $n \geq 3$ and any nonnegative integer m, $\chi_c(\mu_m(K_n)) = \chi(\mu_m(K_n)) = n + 1$. This answers half of the question raised by Zhou in [148] or by Zhu in [145]. In particular, $\mu_m(K_3)$ for arbitrary m is a planar graph with connectivity 3 and maximum degree 4. And this gives another counterexample of a question asked by Vince in [121]. For any positive even number $n(\geq 2)$ and any nonnegative integer m, we show that $\chi_c(\mu_m(K_n)) = n + \frac{1}{t}$, where $t = \lfloor \frac{2m}{n} \rfloor + 1$, this gives a family of arbitrarily large critical graphs G with high connectivity and small maximum degree for which $\chi_c(G)$ can be arbitrarily close to $\chi(G) - 1$.

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