

MASTER'S THESIS

Preconditioners for linear parabolic optimal control problems

Tsang, Siu Chung

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ABSTRACT

In this thesis, we consider the computational methods for linear parabolic optimal control problems. We wish to minimize the cost functional while fulfilling the parabolic partial differential equations (PDE) constraint. This type of problems arises in many fields of science and engineering. Since solving such parabolic PDE optimal control problems often lead to a demanding computational cost and time, an effective algorithm is desired. In this research, we focus on the distributed control problems. Three types of cost functional are considered: Target States problems, Tracking problems, and All-time problems. Our major contribution in this research is that we developed a preconditioner for each kind of problems, so our iterative method is accelerated.

In chapter 1, we gave a brief introduction to our problems with a literature review. In chapter 2, we demonstrated how to derive the first-order optimality conditions from the parabolic optimal control problems. Afterwards, we showed how to use the shooting method along with the *flexible generalized minimal residual* to find the solution. In chapter 3, we offered three preconditioners to enhance our shooting method for the problems with symmetric differential operator. Next, in chapter 4, we proposed another three preconditioners to speed up our scheme for the problems with non-symmetric differential operator. Lastly, we have the conclusion and the future development in chapter 5.

Table of Contents

Declaration	i
Abstract	ii
Acknowledgments	iii
Table of Contents	iv
List of Tables	vii
List of Figures	viii
Chapter 1 Introduction	1
1.1 Background	1
1.2 Motivational Example: Optimal Control of Distributed Heating . . .	3
1.3 Literature Review	4
1.4 Linear Parabolic Optimal Control Problems	6
1.4.1 Target States Problems	7
1.4.2 Tracking Problems	8
1.4.3 All-time Problems	8
Chapter 2 Methodology	10
2.1 Optimality System	10
2.1.1 Discretize then Optimize	11
2.1.2 Optimize then Discretize	15
2.2 Shooting Method	19
2.2.1 Shooting Method for Target States Problems	19

2.2.2	Shooting Method for Tracking Problems and All-time Problems	20
2.2.3	Computation of Shooting Method	21
2.3	Iterative Methods for System of Linear Equations	21
2.3.1	Krylov Subspace Method	22
2.3.2	Preconditioning and FGMRES	23
Chapter 3 Preconditioners for Parabolic Optimal Control Problems with Sym-		
	metric Differential Operator	24
3.1	Preconditioner for Target States Problems with Symmetric Differential	
	Operator	25
3.1.1	Derivation of the Preconditioner	25
3.1.2	Eigenvalues Analysis of the Preconditioned System	28
3.1.3	Computational Cost of P_1^{-1}	30
3.2	Preconditioner for Tracking Problems with Symmetric Differential Op-	
	erator	32
3.2.1	Linear System of Shooting Method	32
3.2.2	Preconditioner and Eigenvalue Analysis	33
3.2.3	Computational Cost of P_2^{-1}	36
3.3	Preconditioner for All-time Problems with Symmetric Differential Op-	
	erator	37
3.3.1	Linear System of Shooting Method	38
3.3.2	Preconditioner and Eigenvalues Analysis	38
3.3.3	Restriction of Parameters	40
3.4	Numerical Result	41
3.4.1	Target States Problems	41
3.4.2	Tracking Problems	42
3.4.3	All-time Problems	43
Chapter 4 Preconditioners for Parabolic Optimal Control Problems with Non-		
	symmetric Differential Operator	47
4.1	Preconditioner for Target States Problems with Non-symmetric Dif-	
	ferntial Operator	47

4.1.1	Derivation of the Preconditioner	48
4.1.2	Computation of the Preconditioner	50
4.2	Preconditioner for Tracking Problems and All-time Problems with Non-symmetric Differential Operator	52
4.2.1	Preconditioner and Engienvalues Analysis	52
4.3	Numerical Result	54
4.3.1	Target States Problems	54
4.3.2	Tracking Problems	54
4.3.3	All-time Problems	56
Chapter 5	Conclusion and Future Development	60
5.1	Conclusion	60
5.2	Future Development	61
	Bibliography	62
	Curriculum Vitae	70