DOCTORAL THESIS

Numerical simulations of the steady Euler equations on unstructured grids

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Numerical Simulations of the Steady Euler Equations on Unstructured Grids

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Abstract

This thesis concerns with effective and robust numerical schemes for solving the steady Euler equations. For solving the nonlinear system resulting from the discretization of the steady Euler equations, we employ a standard Newton method as the outer iterative scheme and a linear multigrid method as the inner iterative scheme with the block lower-upper symmetric Gauss-Seidel iteration as its smoother. The Jacobian matrix of the Newton-iteration is regularized by the local residual, instead of using the commonly adopted time-stepping relaxation technique based on the local CFL number. The local Jacobian matrix of the numerical fluxes are computed by numerical differentiation, which can significantly simplify the implementations by comparing with the manually derived approximate derivatives.

In the reconstruction step, the linear reconstruction and the quadratic reconstruction are studied, respectively. For the linear reconstruction, the approximate polynomial in each cell is obtained by using the WENO reconstruction method. Numerical results demonstrate that the algorithm works very well with the WENO reconstruction. Compared with the results given by using the Venkatakrishnan limiter, the WENO reconstruction method gives superior convergence order, and non-oscillatory and sharp shock profiles. Although the WENO method works very well for the linear case, the convergence to the steady state of the algorithm is affected if the WENO method is extended to the quadratic case directly. So for the quadratic reconstruction, a new hierarchical WENO reconstruction method is introduced to improve the convergence to steady state and also to preserve the formal order of accuracy. Efforts are made to balance the convergence order of the numerical dis-

cretization, the ability of avoiding the non-physical oscillations, and the efficiency of the Newton-iteration.

The last part of the thesis concerns with using the h-adaptive technique to enhance the performance of the proposed numerical algorithms. Numerical results show that, with the h-adaptive methods, the grids around the shock regions are locally refined successfully, which can save a large amount of computational time and memory.

Table of Contents