

DOCTORAL THESIS

Numerical simulations of the steady Euler equations on unstructured grids

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Numerical Simulations of the Steady Euler Equations on Unstructured Grids

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for the degree of
Doctor of Philosophy

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Abstract

This thesis concerns with effective and robust numerical schemes for solving the steady Euler equations. For solving the nonlinear system resulting from the discretization of the steady Euler equations, we employ a standard Newton method as the outer iterative scheme and a linear multigrid method as the inner iterative scheme with the block lower-upper symmetric Gauss-Seidel iteration as its smoother. The Jacobian matrix of the Newton-iteration is regularized by the local residual, instead of using the commonly adopted time-stepping relaxation technique based on the local CFL number. The local Jacobian matrix of the numerical fluxes are computed by numerical differentiation, which can significantly simplify the implementations by comparing with the manually derived approximate derivatives.

In the reconstruction step, the linear reconstruction and the quadratic reconstruction are studied, respectively. For the linear reconstruction, the approximate polynomial in each cell is obtained by using the WENO reconstruction method. Numerical results demonstrate that the algorithm works very well with the WENO reconstruction. Compared with the results given by using the Venkatakrisshnan limiter, the WENO reconstruction method gives superior convergence order, and non-oscillatory and sharp shock profiles. Although the WENO method works very well for the linear case, the convergence to the steady state of the algorithm is affected if the WENO method is extended to the quadratic case directly. So for the quadratic reconstruction, a new hierarchical WENO reconstruction method is introduced to improve the convergence to steady state and also to preserve the formal order of accuracy. Efforts are made to balance the convergence order of the numerical dis-

cretization, the ability of avoiding the non-physical oscillations, and the efficiency of the Newton-iteration.

The last part of the thesis concerns with using the h -adaptive technique to enhance the performance of the proposed numerical algorithms. Numerical results show that, with the h -adaptive methods, the grids around the shock regions are locally refined successfully, which can save a large amount of computational time and memory.

Table of Contents

Declaration	i
Abstract	ii
Acknowledgements	iv
Table of Contents	vi
List of Tables	ix
List of Figures	x
Chapter 1 Introduction	1
1.1 The Euler Equations	2
1.2 The Mesh	5
1.3 The Finite Volume Method	7
1.3.1 Solution Reconstruction	11
1.3.2 Slope Limiter Methods	13
1.3.3 ENO/WENO Reconstruction	14
1.3.4 The HLLC Riemann Solver	16
1.4 The Residual Distribution Method	18
1.5 Temporal Discretization	19
1.6 Some Acceleration Techniques	19
1.6.1 Local Time-Stepping	20
1.6.2 The Multigrid Method	20

1.7	Adaptive Methods	21
1.8	Outline of the Thesis	21
Chapter 2 The Linear Finite Volume Solver		24
2.1	The Finite Volume Discretization for the Steady Euler Equations	27
2.2	The Reconstruction and Limiting Strategies	28
2.2.1	The Linear Reconstruction	29
2.2.2	The Venkatakrishnan Limiter	30
2.2.3	The WENO Reconstruction	33
2.3	Newton Iteration	35
2.3.1	The Jacobian of the Numerical Flux	35
2.3.2	Regularization	36
2.4	Multigrid Method	38
2.4.1	The Projection Operator	39
2.4.2	The Smoother	42
2.5	Boundary Condition	44
2.5.1	Solid Wall Boundary Condition	44
2.5.2	Farfield Boundary Condition	45
2.6	Numerical Results	46
2.6.1	Numerical Convergence Tests	47
2.6.2	The Robustness of the Algorithm	62
2.6.3	Remarks on the Efficiency of the Algorithm	63
2.7	Conclusion Remarks	68
Chapter 3 The Quadratic Solver Based on the Residual Distribution Schemes		69
3.1	High Order Reconstruction	73
3.2	The WENO Hierarchical Limiting Strategy	75
3.2.1	Summarization of the Hierarchical Reconstruction	75

3.2.2	Implementation of the Hierarchical Reconstruction for Steady Problems	78
3.3	Remarks on the Curved Boundary	81
3.4	Numerical Results	84
3.4.1	Numerical Convergence Tests	85
3.4.2	Robustness	99
3.5	Conclusion Remarks	103
Chapter 4	Combination with the Adaptive Techniques	106
4.1	Moving Finite Element Method for the Simulation of Gravity Fingers in Porous Media	107
4.1.1	Finite Element Discretization	108
4.1.2	Moving Mesh Strategy	109
4.1.3	Numerical Results	111
4.2	h -Adaptive Method	114
4.2.1	Mesh Refinement and Coarsening	115
4.2.2	Remarks on the Weight in the Reconstruction	120
4.2.3	Numerical Results	120
4.3	Conclusion Remarks	122
Chapter 5	Concluding Remarks	134
	Bibliography	137
	Curriculum Vitae	147