DOCTORAL THESIS

Application of partial consistency for the semi-parametric models

Zhao, Jingxin

Date of Award:
2017

ABSTRACT

The semi-parametric model enjoys a relatively flexible structure and keeps some of the simplicity in the statistical analysis. Hence, there are abundance discussions on semi-parametric models in the literature. The concept of partial consistency was firstly brought up in Neyman and Scott(1948). It was said the in cases where infinite parameters are involved, consistent estimators are always attainable for those "structural" parameters. The "structural' parameters are finite and govern infinite samples. Since the nonparametric model can be regarded as a parametric model with infinite parameters, then the semi-parametric model can be easily transformed into a infinite-parametric model with some "structural" parameters. Therefore, based on this idea, we develop several new methods for the estimating and model checking problems in semi-parametric models.

The implementation of applying partial consistency is through the method "local average". We consider the nonparametric part as piecewise constant so that infinite parameters are created. The "structural" parameters shall be the parametric part, the model residual variance and so on. Due to the partial consistency phenomena, classical statistic tools can then be applied to obtain consistent estimators for those "structural" parameters. Further more, we can take advantage of the rest of parameters to estimate the nonparametric part. In this thesis, we take the varying coefficient model as the example. The estimation of the functional coefficient is discussed and relative model checking methods are presented.

The proposed new methods, no matter for the estimation or the test, have remarkably lessened the computation complexity. At the same time, the estimators and the tests get satisfactory asymptotic statistical properties. The simulations we conducted for the new methods also support the asymptotic results, giving a relatively efficient and accurate performance. What's more, the local average method is easy to understand and can be flexibly applied to other type of models. Further developments could be done on this potential method.

In Chapter 2, we introduce a local average method to estimate the functional coefficients in the varying coefficient model. As a typical semi-parametric model, the

varying coefficient model is widely applied in many areas. The varying coefficient model could be seen as a more flexible version of classical linear model, while it explains well when the regression coefficients do not stay constant. In addition, we extend this local average method to the semi-varying coefficient model, which consists of a linear part and a varying coefficient part. The procedures of the estimations are developed, and their statistical properties are investigated. Plenty of simulations and a real data application are conducted to study the performance of the proposed method.

Chapter 3 is about the local average method in variance estimation. Variance estimation is a fundamental problem in statistical modelling and plays an important role in the inferences in model selection and estimation. In this chapter, we have discussed the problem in several nonparametric and semi-parametric models. The proposed method has the advantages of avoiding the estimation of the nonparametric function and reducing the computational cost, and can be easily extended to more complex settings. Asymptotic normality is established for the proposed local average estimators. Numerical simulations and a real data analysis are presented to illustrate the finite sample performance of the proposed method.

Naturally, we move to the model checking problem in Chapter 4, still taking varying coefficient models as an example. One important and frequently asked question is whether an estimated coefficient is significant or really "varying". In the literature, the relative hypothesis tests usually require fitting the whole model, including the nuisance coefficients. Consequently, the estimation procedure could be very compute-intensive and time-consuming. Thus, we bring up several tests which can avoid unnecessary functions estimation. The proposed tests are very easy to implement and their asymptotic distributions under null hypothesis have been deduced. Simulations are also studied to show the properties of the tests.

Keywords: Partial consistency; Local average; Varying coefficient model; Variance estimation; Model checking; Model fitting.

Table of Contents

Declaration i

Abstract ii

Acknowledgements iv

Table of Contents v

List of Tables viii

List of Figures ix

Chapter 1 Introduction 1

1.1 Semi-parametric model 2

1.2 Statistical inferences for the semi-parametric model 3

1.2.1 Model fitting 3

1.2.2 Variance estimation 4

1.2.3 Model checking 6

1.3 Partial consistency and local average 7

1.4 Outline of the thesis 8

Chapter 2 Local Average Fitting in Varying Coefficient Model 10

2.1 Introduction 10

2.2 Methodology 13

2.2.1 Varying coefficient model 13

2.2.2 Semi-varying coefficient model 15

2.2.3 local average method 17

2.3 Theorem 20
2.4 Application and Simulation 23
2.4.1 A simple application 23
2.4.2 Simulation for varying coefficient model 26
2.4.3 Simulation for semi-varying coefficient model 30
2.5 Concluding remarks 34
2.6 Appendix 35

Chapter 3 Variance Estimation for Semi-parametric Regression Models 40
3.1 Introduction 40
3.2 Variance estimation by local average 43
3.2.1 Review of classical methods 43
3.2.2 Local average method 45
3.2.3 Theoretical properties 46
3.3 Extensions of the estimator 47
3.3.1 Partially linear models 47
3.3.2 Varying coefficient models 48
3.3.3 Refined local average variance estimator 50
3.3.4 More extensions 52
3.4 Applications of local average variance estimation 54
3.4.1 Confidence interval of variance estimation 54
3.4.2 Nonparametric hypothesis testing 55
3.4.3 Variance function estimation 56
3.5 Numerical studies 57
3.5.1 Simulations for variance estimation 57
3.5.2 Applications of variance estimation 67
3.5.3 Real data analysis 70
3.6 Conclusion and discussion 72
3.7 Appendix 73

Chapter 4 Hypothesis Testing in Varying Coefficient Models 79
4.1 Introduction 79

4.2 Methodology 82
4.3 Theorem 89
4.4 Simulation 92
4.5 Concluding remarks 97
4.6 Appendix 99

Bibliography 112

Curriculum Vitae 118