

## DOCTORAL THESIS

### Lax representations, Hamiltonian structures, infinite conservation laws and integrable discretization for some discrete soliton systems

Zhu, Zuonong

*Date of Award:*  
2000

[Link to publication](#)

#### General rights

Copyright and intellectual property rights for the publications made accessible in HKBU Scholars are retained by the authors and/or other copyright owners. In addition to the restrictions prescribed by the Copyright Ordinance of Hong Kong, all users and readers must also observe the following terms of use:

- Users may download and print one copy of any publication from HKBU Scholars for the purpose of private study or research
- Users cannot further distribute the material or use it for any profit-making activity or commercial gain
- To share publications in HKBU Scholars with others, users are welcome to freely distribute the permanent URL assigned to the publication

**Lax Representations, Hamiltonian Structures, Infinite  
Conservation Laws and Integrable Discretization  
for Some Discrete Soliton Systems**

ZHU Zuonong

A thesis submitted in partial fulfilment of the requirements  
for the degree of  
Doctor of Philosophy

January 2000

Hong Kong Baptist University

# Abstract

---

The purpose of this thesis is to explore the integrability of some discrete soliton systems, such as the Lax representations, Hamiltonian structures, infinite conservation laws, integrable discretizations, and so on.

From proper discrete spectral problems, new integrable lattice systems are proposed by semi-discrete zero curvature equation. The new lattice systems obtained contain many known integrable lattice equations, such as the discrete NLS equation, the discrete KdV-type equations, the Toda-type lattice equations, the Volterra-type lattice equations, the Blaszak-Marciniak lattice, the Suris lattices, and so on.

We obtain the Hamiltonian structures for some integrable lattice systems by the trace identity method. The conservation laws of the Toda-type and the Volterra-type lattice systems are also given.

Integrable systems with both spatial and temporal discretizations play an increasingly important role in the modern developments of integrable system theory. In this thesis we also focus on the integrable discretization of the new integrable lattice systems. Integrable discretizations of the general Toda-type lattice equation, the relativistic Volterra lattice equation, the general Blaszak-Marciniak lattice equation and some other integrable lattice systems are obtained. The Lagrangian and Newtonian forms of integrable discretizations of the Toda-

type lattice equations which occur in the literatures are given uniformly and some new integrable discretizations of the Toda-type lattice are proposed.

# Contents

---

|  |           |
|--|-----------|
| Declaration.....   | i         |
| Abstract.....  | ii        |
| Acknowledgements.....  | iv        |
| <b>1 Introduction.....</b>   | <b>1</b>  |
| <b>2 Some discrete integrable hierarchies.....</b>   | <b>11</b> |
| 2.1 Integrable lattice hierarchies associated with<br>isospectral problem (1.34).....              | 12        |
| 2.2 Reductions of two integrable lattice hierarchies.....  | 16        |
| 2.3 Integrable lattice hierarchies for nonisospectral problem (1.34).....                          | 21        |
| 2.4 Integrable lattice systems associated with spectral problem (1.35).....                        | 25        |
| 2.5 Integrable lattice system associated with isospectral problem (1.36).....                      | 37        |
| 2.6 A general Blaszk-Marciniak integrable lattice system.....                                      | 40        |
| <b>3 Lax pairs for known integrable lattice equations and<br/>discrete KdV-type equations.....</b> | <b>46</b> |
| 3.1 Lax pairs for known integrable lattice equations.....  | 46        |
| 3.2 Discrete KdV-type equations.....   | 57        |
| <b>4 Hamiltonian structures of integrable lattice systems.....</b>                                 | <b>66</b> |
| <b>5 Conservation laws of the Toda-type and Volterra-type equations....</b>                        | <b>73</b> |
| 5.1 Miura map between Toda-type lattices and Volterra-type lattices....                            | 73        |
| 5.2 Conservation laws of Volterra-type lattices and Toda-type lattices....                         | 75        |
| <b>6 Integrable time discretizations of integrable lattice systems.....</b>                        | <b>83</b> |
| 6.1 Integrable discretization of the general Toda-type lattice equation....                        | 83        |

6.2 Integrable discretization of the relativistic Volterra lattice.....93

6.3 Integrable discretization of two lattice systems  
and full discretizations of continuous soliton equations.....96

6.4 Integrable discretization of the general BM lattice system.....108

6.5 Integrable discretizations of lattice systems (2.78) and (2.123)..... 113

**Bibliography**..... 118

**Curriculum Vitae**..... 125