DOCTORAL THESIS

Lax representations, Hamiltonian structures, infinite conservation laws and integrable discretization for some discrete soliton systems

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Abstract

The purpose of this thesis is to explore the integrability of some discrete soliton systems, such as the Lax representations, Hamiltonian structures, infinite conservation laws, integrable discretizations, and so on.

From proper discrete spectral problems, new integrable lattice systems are proposed by semi-discrete zero curvature equation. The new lattice systems obtained contain many known integrable lattice equations, such as the discrete NLS equation, the discrete KdV-type equations, the Toda-type lattice equations, the Volterra-type lattice equations, the Blaszak-Marciniak lattice, the Suris lattices, and so on.

We obtain the Hamiltonian structures for some integrable lattice systems by the trace identity method. The conservation laws of the Toda-type and the Volterra-type lattice systems are also given.

Integrable systems with both spatial and temporal discretizations play an increasingly important role in the modern developments of integrable system theory. In this thesis we also focus on the integrable discretization of the new integrable lattice systems. Integrable discretizations of the general Toda-type lattice equation, the relativistic Volterra lattice equation, the general Blaszak-Marciniak lattice equation and some other integrable lattice systems are obtained. The Lagrangian and Newtonian forms of integrable discretizations of the Toda-

type lattice equations which occur in the literatures are given uniformly and some new integrable discretizations of the Toda-type lattice are proposed.

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