DOCTORAL THESIS

Preconditioning techniques for all-at-once linear systems arising from advection diffusion equations

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Abstract

In this thesis, we mainly study preconditioning techniques for all-at-once linear systems arising from discretization of three types of time-dependent advection-diffusion equation: linear diffusion equation, constant-coefficients advection-diffusion equation, time-fractional sub-diffusion equation. The proposed preconditioners are used with Krylov subspace solvers.

The preconditioner developed for linear diffusion equation is based on ϵ-circulant approximation of temporal discretization. Diagonalizability, clustering of spectrum and identity-plus-low-rank decomposition are derived for the preconditioned matrix. We also show that generalized minimal residual (GMRES) solver for the preconditioned system has a linear convergence rate independent of matrix-size.

The preconditioner for constant-coefficients advection-diffusion equation is based on approximating the discretization of advection term with a matrix diagonalizable by sine transform. Eigenvalues of the preconditioned matrix are proven to be lower and upper bounded by positive constants independent of discretization parameters. Moreover, as the preconditioner is based on spatial approximation, it is also applicable to steady-state problem. We show that GMRES for the preconditioned steady-state problem has a linear convergence rate independent of matrix size.

The preconditioner for time-fractional sub-diffusion equation is based on approximating the discretization of diffusion term with a matrix diagonalizable by sine transform. We show that the condition number of the preconditioned matrix is bounded by a constant independent of discretization parameters so that the normalized conjugate gradient (NCG) solver for the preconditioned system has a linear convergence rate independent of discretization parameters and matrix size.

Fast implementations based on fast Fourier transform (FFT), fast sine transform (FST) or multigrid approximation are proposed for the developed preconditioners. Numerical results are reported to show the performance of the developed precondi-

tioners.

Keywords: Advection-diffusion equation; Preconditioner; Toeplitz matrix; Convergence of iterative solver; Condition number

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