MASTER'S THESIS

Estimation of the reciprocal of a binomial proportion

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Abstract

As a classic parameter originated from the binomial distribution, the binomial proportion has been well studied in the literature due to its wide range of applications. In contrast, the reciprocal of the binomial proportion, also known as the inverse proportion, is often overlooked, although it plays an important role in sampling designs and clinical studies. To estimate the inverse proportion, a simple method is to apply the maximum likelihood estimation (MLE). This estimator is, however, not a valid estimator because it suffers from the zero-event problem, which occurs when there is no successful event in the trials. At first, we review a number of methods proposed to overcome the zero-event problem and discuss whether they are feasible to estimate the inverse proportion. Inspired by the Wilson (1927) and Agresti and Coull (1998), in this thesis, we focus on a family of shrinkage estimators of the inverse proportion and propose to derive the optimal estimator within this family. The shrinkage estimator overcomes the zero-event problem by including a positive shrinkage parameter, which is intrinsically related to the expected value of the resulting estimator. To find the best shrinkage parameter, the relationship between the shrinkage parameter and the estimation bias of the shrinkage estimator is investigated systematically. Note that the explicit expression of the expected value function of the estimator and the best shrinkage parameter are quite complicated to compute when the number of trials is large. Hence, we review three methods in the literature which were proposed to approximate the expected value function. And after being inspired, we propose a new approximate formula for the expected value function and derive an approximate solution of the optimal shrinkage parameter by the Taylor expansion. Because there still exist an unknown binomial proportion in the optimal shrinkage parameter, we suggest a plug-in estimator for the unknown proportion with an adaptive threshold. Finally, simulation studies are conducted to evaluate the performance of our new estimator. As baselines for comparison, we also include the Fattorini estimator, the Haldane estimator and a piecewise estimator in the simulations. According to the simulation results, the new estimator is able to achieve a better or equally good performance compared with the Fattorini estimators in most settings. Hence, our new estimator can be a reliable estimator for the inverse proportion in most practical cases.

Key words: Binomial proportion, Estimation bias, Inverse proportion, Plug-in estimator, Shrinkage parameter, Zero-event problem

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