

MASTER'S THESIS

On the classification and selection of orthogonal designs

Weng, Lin Chen

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Abstract

Factorial design has played a prominent role in the field of experimental design because of its richness in both theory and application. It explores the factorial effects by allowing the arrangement of efficient and economic experimentation, among which orthogonal design, uniform design and some other factorial designs have been widely used in various scientific investigations. The main contribution of this thesis shows the recent advances in the classification and selection of orthogonal designs.

Design isomorphism is essential to the classification, selection and construction of designs. It also covers various popular design criteria as necessary conditions, such connection has led to a rapid growth of research on the novel approaches for either detecting the non-isomorphism or identifying the isomorphism. But further classification of non-isomorphic designs has received little attention, and hence remains an open question. It motivates us to propose the degree of isomorphism, as a more general view of isomorphism, for classifying non-isomorphic subclasses in orthogonal designs, and develop the column-wise identification framework accordingly.

Selecting designs in sequential experiments is another concern. As a well-recognized strategy for improving the initial design, fold-over techniques have been widely applied to construct combined designs with better property in a certain sense. While each fold-over method has been comprehensively studied, there is no discussion on the comparison of them. It is the motivation behind our survey on the existing fold-over methods in view of statistical performance and computational complexity.

The thesis involves five chapters and it is organized as follows. In the beginning chapter, the underlying statistical models in factorial design are demonstrated. In particular, we introduce orthogonal design and uniform design associated with commonly-used criteria of aberration and uniformity.

In Chapter 2, the motivation and previous work of design isomorphism are reviewed. It attempts to explain the evolution of strategies from identification methods to detection methods, especially when the superior efficiency of the latter has been gradually appreciated by the statistical community.

In Chapter 3, the concepts including the degree of isomorphism and pairwise distance are proposed. It allows us to establish the hierarchical clustering of non-isomorphic orthogonal designs. By applying the average linkage method, we present a new classification of $\mathcal{L}_{27}(3^{13})$ with six different clusters.

In Chapter 4, an efficient algorithm for measuring the degree of isomorphism is developed, and we further extend it to a general framework to accommodate different issues in design isomorphism, including the detection of non-isomorphic designs, identification of isomorphic designs and the determination of non-isomorphic subclass for unclassified designs.

In Chapter 5, a comprehensive survey of the existing fold-over techniques is presented. It starts with the background of these methods, and then explores the connection between the initial designs and their combined designs in a general framework. The dictionary cross-entropy loss is introduced to simplify a class of criteria that follows the dictionary ordering from pattern into scalar, it allows the statistical performance to be compared in a more straightforward way with visualization.

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