

## DOCTORAL THESIS

### On the construction of uniform designs and the uniformity property of fractional factorial designs

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# Abstract

Uniform design has found successful applications in manufacturing, system engineering, pharmaceuticals and natural sciences since it appeared in 1980's. Recently, research related to uniform design is emerging. Discussions are mainly focusing on the construction and the theoretical properties of uniform design. On one hand, new construction methods can help researchers to search for uniform designs in more efficient and effective ways. On the other hand, since uniformity has been accepted as an essential criterion for comparing fractional factorial designs, it is interesting to explore its relationship with other criteria, such as aberration, orthogonality, confounding, etc.

The first goal of this thesis is to propose new uniform design construction methods and recommend designs with good uniformity. A novel stochastic heuristic technique, the adjusted threshold accepting algorithm, is proposed for searching uniform designs. This algorithm has successfully generated a number of uniform designs, which outperforms the existing uniform design tables in the website "<https://uic.edu.hk/~isci/UniformDesign/UD%20Tables.html>". In addition, designs with good uniformity are recommended for screening either qualitative or quantitative factors via a comprehensive study of symmetric orthogonal designs with 27 runs, 3 levels and 13 factors. These designs are also outstanding under other traditional criteria.

The second goal of this thesis is to give an in-depth study of the uniformity property of fractional factorial designs. Close connections between different criteria and lower bounds of the average uniformity have been revealed, which can be used as benchmarks for selecting the best designs. Moreover, we find non-isomorphic designs have different combinatorial and geometric properties in their projected and level permuted designs. Two new non-isomorphic detection methods are proposed

and utilized for classifying fractional factorial designs. The new methods take advantages over the existing ones in terms of computation efficiency and classification capability. Finally, the relationship between uniformity and isomorphism of fractional factorial designs has been discussed in detail. We find isomorphic designs may have different geometric structure and propose a new isomorphic identification method. This method significantly reduces the computational complexity of the procedure. A new uniformity criterion, the uniformity pattern, is proposed to evaluate the overall uniformity performance of an isomorphic design set.

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