DOCTORAL THESIS

Complete spatial randomness tests, intensity-dependent marking and neighbourhood competition of spatial point processes with applications to ecology

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Complete Spatial Randomness Tests, Intensity-dependent Marking and Neighbourhood Competition of Spatial Point Processes with Applications to Ecology

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ABSTRACT

Point pattern analysis is a major area of spatial statistics. An important task of the analysis of spatial point patterns is to check how well a model fits the observed data. Of particular interest is whether a point pattern is completely spatially random (CSR), i.e. a realization of a stationary Poisson process or a binomial point process. A typical way to test the CSR hypothesis is to compare a summary function, such as the nearest neighbor distance distribution function, of a given point pattern with the theoretical form under the CSR hypothesis. However, the estimation of a summary function is intervened by edge-effects because point patterns are observed via a bounded sampling window. Various edge-corrected estimators for different summary functions have been proposed in the literature.

Many authors investigated the power of different edge-corrected estimators of a summary function, generically denoted by H in testing the CSR by using the test statistic

$$\max_{r \leq r_0} \left| H(r) - \hat{H}(r) \right|,$$

where r_0 is a suitably chosen upper limit. The choice of r_0 sometimes is crucial for testing the CSR because typically the variance of $\hat{H}(r)$, an edge-corrected estimator of $H(r)$, increases as r increases. Some authors suggested that r_0 should be 0.25 for 25 points and 0.125 for 100 points observed in a unit square on $\mathbb{R}^2$. In this thesis, three approaches were suggested to make the choice of r_0 less crucial, namely,

1. adding a weight function to the difference,

2. using a recently proposed adapted intensity estimator to stabilize the variance,

3. testing uniformity of a spatial point pattern.

We applied these approaches to simulated data and real data, and found that the first approach, though improves the power of the test, still depends on r_0; the second approach does make r_0 less crucial but still cannot eliminate it; the third approach is totally free from r_0 and would be recommended.

Marked point processes become more and more interesting in spatial statistics. They represent a natural approach to the analysis of data where at random positions

random variables are observed. This is typical for biological applications where the 'points' are the locations of plants (trees) and the 'marks' plant attributes. Here the application of the classical methods of point process statistics is not sufficient, because we want to analyze not only the points but also the correlation between marks and between marks and points. To allow correlations between points and marks, we introduced a suitable model with a close relationship between point density and marks. Characteristics of the new model were derived and used to analyze real data.

Furthermore, we considered the neighborhood competition in a forest using the notion of a marked point process by two existing approaches, and introduced two new summary characteristics to describe the global competition situation in a forest.

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