

## DOCTORAL THESIS

### Stochastic optimal control in randomly-branching environments

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**Stochastic Optimal Control  
in Randomly-branching Environments**

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A thesis submitted in partial fulfillment of the requirements  
for the degree of  
Doctor of Philosophy

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## **Abstract**

In this research, a framework is proposed to formalize optimization situations with stochastic dynamics and uncertain environments over time in the following manner. Since the future environment of the problem are not known with certainty, the concept of “random furcating” is introduced to describe future payoffs which change at any instant of time according to known probability distributions, which turn are defined in terms of multiple-branching stochastic processes. The introduction of this stochastic specification lead to a novel approach to solve dynamic control in terms of properties and solution concepts not explored in the previous literature. New and significant mathematical results are obtained, under which it becomes possible to characterize the conditions under which previously unsolvable control problems can be solved.

In addition to a new and analytically interesting way to model control situations over time, our research widens the scope of control theory to real world problems. In particular, this new approach widens the application of dynamic control theory to problems where future environments are not known with certainty.

The particular interest are solution techniques for randomly furcating endogenous horizon optimal control, infinite-time randomly furcating optimal control with infinite overlapping generations randomly furcating optimal control. Important applications

abound in economics and resource pricing. A class of resource extraction problems involving stochastic dynamics and randomly fluctuating non-autonomous payoffs are developed. A general solution mechanism is characterized and computer algorithms for solving the exact solution are developed. Exact solution to this stochastically complicated problem is presented.

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